CONDITIONS OF EQUILIBRIUM

Introduction

Aim: To investigate the conditions required for an object to be in equilibrium

This exercise looks at a rigid object which is in both translational and rotational equilibrium and is designed to help you:

- determine the resultant force on an object from a scale vector diagram.
- determine the resultant force on an object by resolving the forces into orthogonal components.

What To Do Before You Get To Lab

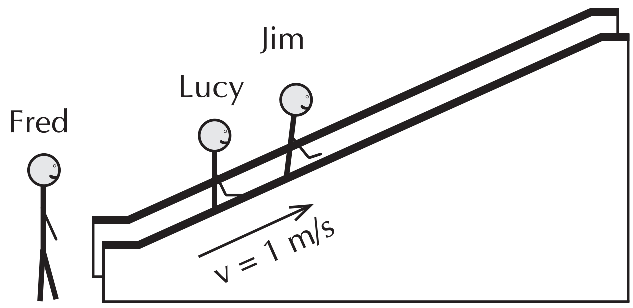
1) Read through the preliminary information and complete the preliminary exercises below
2) Read through the rest of the exercise, so that you will know what to do in the laboratory.

Preliminary Information

Motion and Reference Frames

The simplest type of motion an object can have is ***translational*** (or linear) motion, i.e. motion in a straight line. To describe motion you need to have a ***frame of reference*** , this is a coordinate system that you set up to allow you to explain how fast and far an object has moved, relative to something else.

Example: Lucy is standing on an escalator travelling at 1 m/s relative to the ground. Jim runs past her and Lucy measures his speed as 2 m/s relative to herself. However, Fred is standing still at the bottom of the escalator so he sees Jim run up the escalator at 1 + 2 = 3 m/s. Each person measures their speed in a different frame of reference.



Exercise: Complete the table below which shows the speed of each person in the three reference frames.

	Fred's Reference Frame	Lucy's Reference Frame	Jim's reference frame
Fred's speed			
Lucy's speed	1 m/s	0 m/s	-2 m/s
Jim's speed			

Changing the state of motion
When external forces act on a rigid object it will change its "state of motion." A change in the state of motion means that the object is accelerating, this can happen in two ways:

1. *Translational* acceleration: the velocity of its centre of mass is changing
2. *Rotational* acceleration: the speed of rotation is changing

Equilibrium
This experiment looks at rigid body which has forces acting on it but is in equilibrium. An object is in equilibrium when its state of motion is not changing. There are two cases of equilibrium:

1. *Translational* equilibrium: the velocity of the object is not changing
2. *Rotational* equilibrium: the speed of rotation of the object is not changing

If an object is in equilibrium the sum of all the forces acting on it equal to zero. To study the effect of forces on objects we need to use vectors.

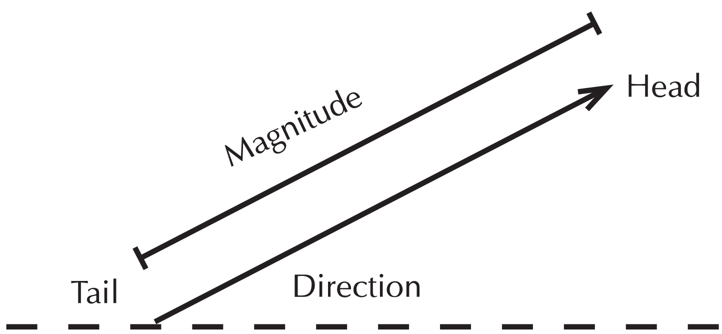
Scalars and Vectors
A ***scalar*** is a quantity which is described completely by its ***magnitude***. For example temperature, pressure, distance and speed. Scalars add algebraically, for example if you walk 10 km then 5 km , the total distance you have walked is 10 + 5 = 15 km

A ***vector*** is a quantity which is described by it ***magnitude*** and ***direction***. For example, displacement , velocity and force. Vectors do not add algebraically because the direction needs to be taken into account. Using a simple example, if you walked 10 km, then turned around 180 degrees and walked back 5km, your *total displacement* is 10 - 5 = 5km from where you started (even though you have walked a *distance* of 15 km) . We'll now look at two difference ways of adding vectors together.

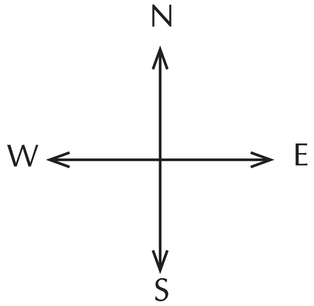
Using Scale Vector Diagrams to Add Vectors Together

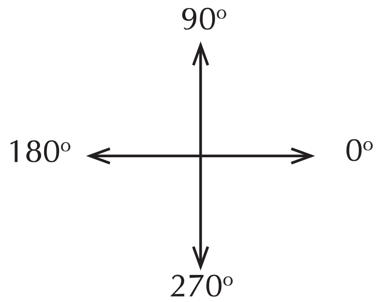
In a scale vector diagram:

- Vectors are represented by *arrows* showing the *direction*
- The *length* of the line gives the *magnitude* of the vector, according a scale
- All vectors have a ***head (arrow)*** and a ***tail (no arrow)***

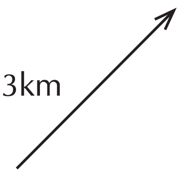


When you are drawing vectors you need to also draw the reference frame you are using to show the direction of the vectors, here are some examples





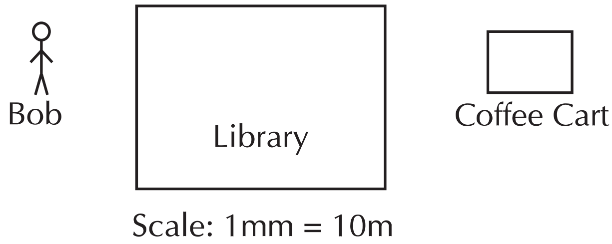
Exercise: Complete the table below, using the above reference frames

Vector	*Scale*	*Picture*
A dog walks 3 km, NE	1 cm = 1 km Therefore the length of the line will be 3cm in this case	3km 
A man runs 400m at 196°	1cm = 100 m	a) Draw the picture
A plane flies 2 000 000 m at 20° south of east	b) Calculate the scale	c) Draw the picture

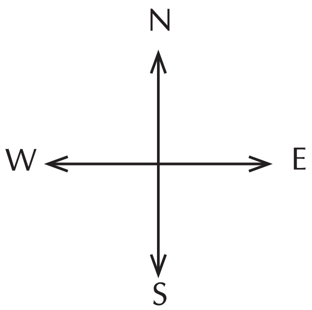
Adding vectors

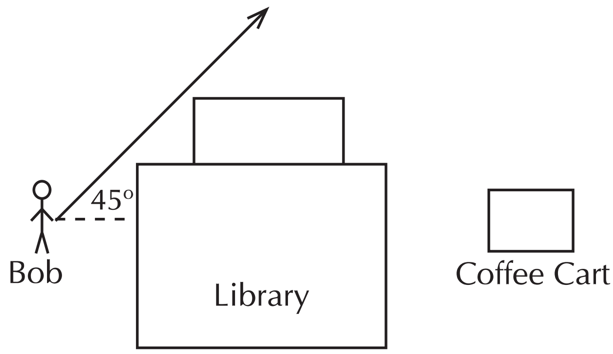
Scale vector diagrams can be used to ***add vectors*** together by drawing the arrows ***"head to tail"***. The ***resultant vector*** is then given by drawing another vector from the ***tail of the first vector to the head of the last vector***. A resultant vector is a single vector which would have the same effect as all other vectors added together.

Example: Bob the student wants to walk to the coffee cart which is east from him, however between Bob and the coffee cart is the library...Bob is avoiding the library because he has some large fines to pay so he decides to walk around it.

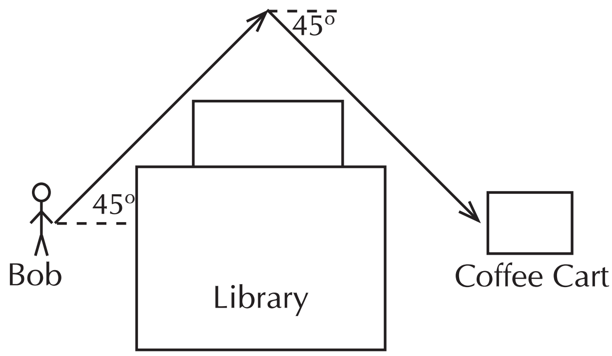


He walks NE for 40m
(we draw a vector 40mm long, using the scale 1mm = 10m, in NE direction)

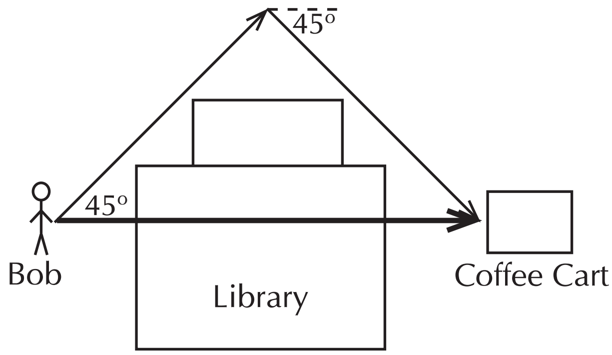




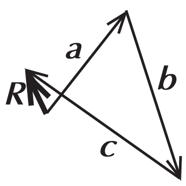
Then he walk SE for 40m (we now draw another vector 40mm long starting at the head of the last vector)



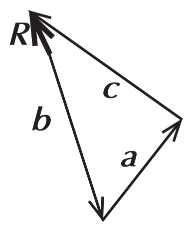
The resultant vector is drawn from the tail of the first vector to the head of the last vector. The resultant vector is equal to the two other vectors added together. By measuring the length of the resultant vector we find it is 56.5 mm long, using the scale this is converted to 56.5m and the direction is east. Therefore if Bob had walked through the library he would have walked 56.5 m east.
i.e. 40m NE + 40m SE = 56.6m E



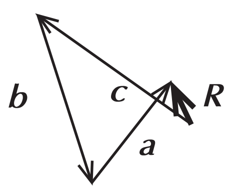
Order of vectors
You can add vectors in any order as long as you add them head to tail



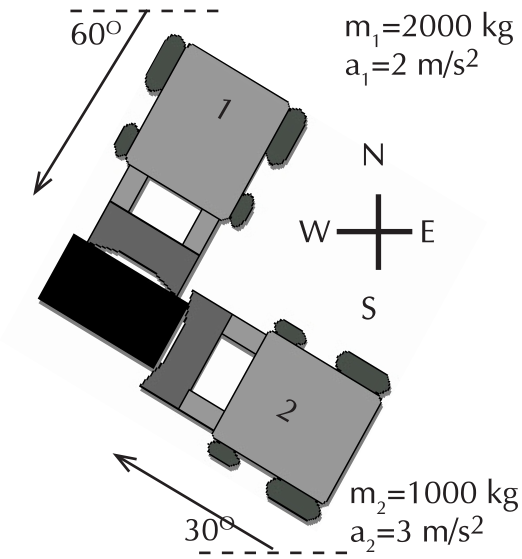
$\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{R}$



$\mathbf{b} + \mathbf{a} + \mathbf{c} = \mathbf{R}$



$\mathbf{c} + \mathbf{b} + \mathbf{a} = \mathbf{R}$



Exercise

Work through this exercise to practice drawing vectors.

Two bulldozers are used to push a block of concrete, as shown in the diagram. Use a scale vector diagram to work out the resultant force on the block.

a) Fill in the units and the second row of the table below to work out the force that each tractor exerts on the block.

force = mass x acceleration

Tractor	Mass	Acceleration	Force- magnitude	Force - direction
units				
1	2000	2	4000	60 S of W
2				

b) Choose an appropriate scale for the diagram.

Scale:

c) Draw the first vector in the space below using a ruler and protractor

d) Draw the second vector, starting at the tail of the first vector

e) Draw the ***resultant*** vector from the ***tail*** of the first vector to the ***head*** of the second vector

f) Measure the length of the resultant vector and use the scale to convert it to newtons

Length = Magnitude =

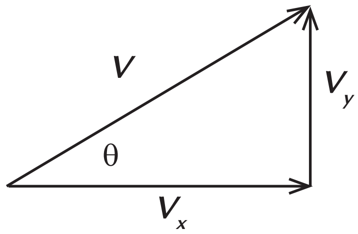
g) Measure the direction of the resultant vector

Direction =

Resolving Vectors Into Components

Another way to solve vector problems is to resolve a vector into horizontal and vertial components.

Example: A vector $\boldsymbol{V}$ has magnitude V and direction θ and $\boldsymbol{V} = \boldsymbol{V}_x + \boldsymbol{V}_y$,
where $\boldsymbol{V}_x$ is a horizontal vector and $\boldsymbol{V}_y$ is a vertical vector.
Using trigonometry,



$V_x = V \cos \theta$ $\qquad$ $V_y = V \sin \theta$

$V^2 = V_x^{\,2} + V_y^{\,2}$ $\qquad$ $\theta = \tan^{-1}(V_y/V_x)$

Once vectors have been resolved into their components
we can add them by just adding the components, then use trigonometry to work out the final answer.

Exercise: Solve the previous problem by resolving the vectors into their components

Tractor 1 $\qquad$ Tractor 2

a) For the second vector, calculate the x and y components, using trigonometry.

Tractor	Direction	Magnitude (N)	x component (N)	x direction (+ or -)	y component (N)	y direction (+ or -)
T1	60° S of W	4000	=4000 x cos 60 = 2000	-	=4000 x sin 60 = 3464	-
T2						

b) Add the x components of the vectors together and add the y components together

Resultant x = tractor 1 x + tractor 2 x

$R_x = T1_x + T2_x$

=

=

Resultant y = tractor 1 y + tractor 2 y

$R_y = T1_y + T2_y$

=

=

c) Write down an equation to find the magnitude of the resultant force R and solve it

Magnitude=

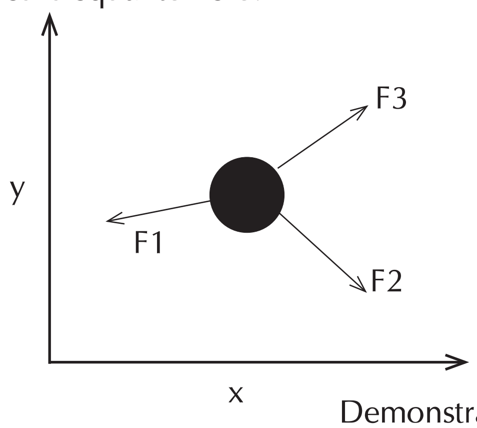
d) Write down an equation to find the direction of the resultant force and solve it

Direction =

e) Compare this answer to your previous answer

Exercise: Forces F1, F2 and F3 are all acting on a disc in a x-y plane. The forces are in translational equilibrium, i.e. the sum of all the forces is equal to zero.

a) Express this condition as an equation.



b) The forces can be resolved into their x and y components. Write down equations to show the equilibrium condition in each direction.

Demonstrator:

Discussion with Demonstrator

Your demonstrator will discuss this section with your group

Stages of a Typical Experiment*

Stage	***What it includes and the benefits***
Aim	What do you want to find out?
The plan	· Equipment · Methods · Theory
Preparation	· Organise the experiment · Collect experiment · Learn how to use equipment
Preliminary Experiment	· Practice using equipment · Indicates which features work well · What values to expect
Collecting data	· Drawing up tables to collect data · Including units · Making sufficient measurements · Repeat Experiment
Analysis of data	· Calculations · Graphs · Relationship to theory
Conclusions	· Have you answered your aim? · Important results · What do the results mean · How can you improve the experiment · Sources of errors and uncertainties

* Les Kirkup; Experimental Methods, An Introduction to the Analysis and Presentation of Data, John Wiley & Sons 1994

Significant figures and rounding values

· A simple measurement, especially with an automatic device such as a calculator, may return a value with many figures that include some non-meaningful figures.

· You need to decide how many of the figures in your final answer are significant.

· Avoid rounding numbers during calculations

· Present rounded values for intermediate and final results

· Your final quoted errors should have no more than two significant errors

Exercise to answer as a group

Raw Data	Raw final answer	Number of significant figures you should quote
5.6, 8.51	0.6462984	
3400, 2300, 1500	6749.32	
0.00021, 0.00034, 0.000654	0.0000321	
56.0, 84	3	

Significant Figures Rule: The number of significant figures in your final answer should be:

____

____

Demonstrator:

Equipment

You will need the following:

- a special vertical "force board" apparatus with clips (see picture below)
- 5 mass carriers carrying 4 masses each
- a small, flat, irregularly shaped brass plate with 5 strings attached
- a small plane mirror
- a protractor
- a sheet of A3 paper

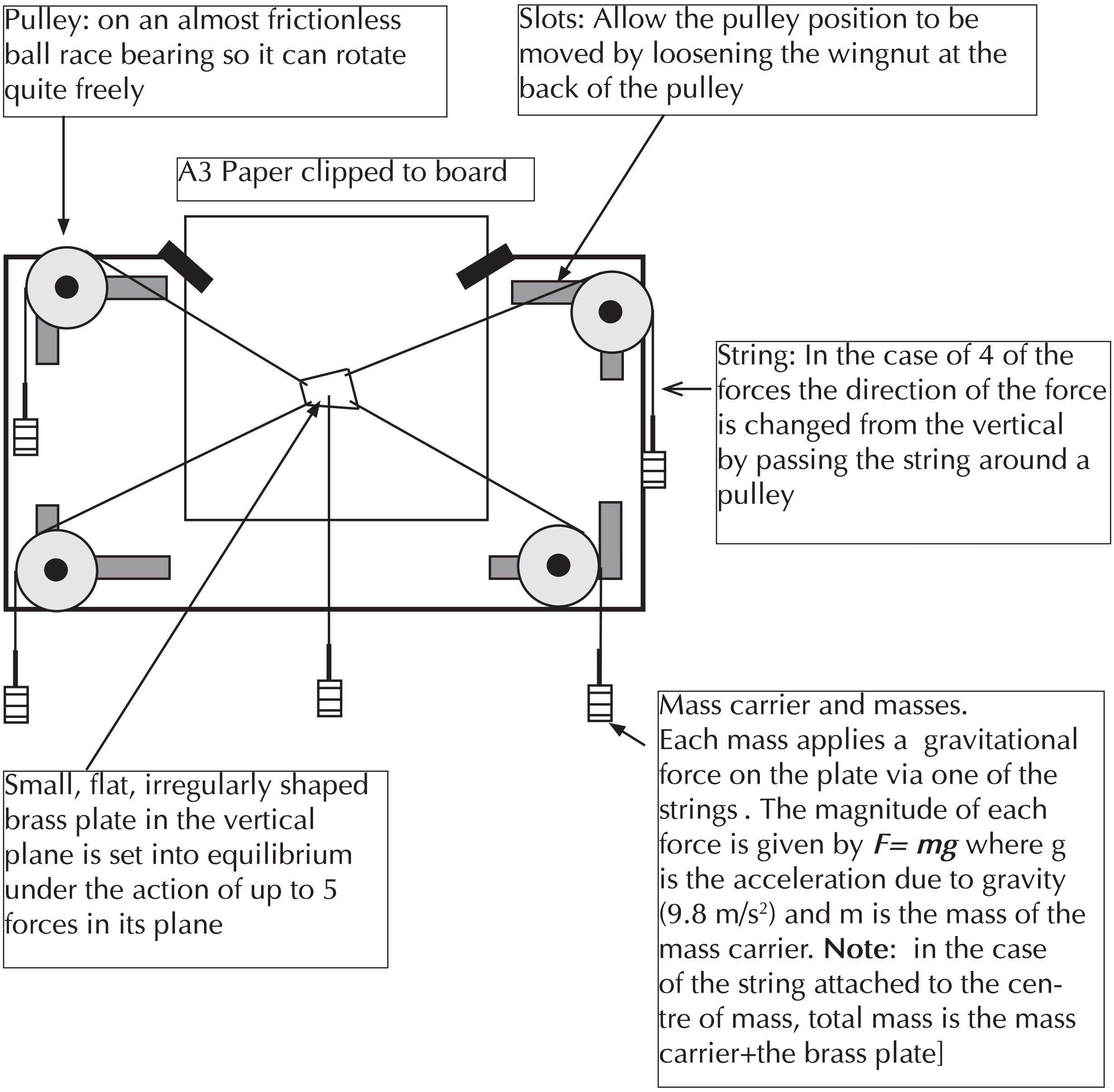


Figure A: The Force Board

In the Laboratory

Obtain the equipment from the shelves and cupboards. Check it against the list above to make sure everything is there. If anything is missing, see one of the lab attendants.

Part 1. Preliminaries

1. Use one of the mass balances in the laboratory to determine the mass of the small, flat, brass plate.

Mass of the brass plate = ______________________

Remember to write the units!

2. Set up your vertical force board as shown in figure A. Start out with only the top two strings loaded with mass carriers, and the other strings hanging loosely. Then add the other mass carriers to the ends of each string one by one.

WARNING
Be very careful not to allow any of the mass carriers to drop to the floor.

3. Once you have all the mass carriers in place, adjust the locations of the pulleys, and the amount of mass on the suspended mass carriers until the plate is approximately in the centre of the board.

The final mass of each mass carrier + masses should greater than 0.200 kg. Some will need a much larger mass.

Why should the final masses all be at least 0.200 kg?

Part 2. Translational Equilibrium of the Plate

You need to obtain an "as accurate as possible" replica of the outline and position of the plate, and the directions and magnitude of the forces acting on it

1. Use the clips provided on the top of the force board to suspend the A3 page against the board behind the plate and strings, so that the plate is (roughly) in the middle of the page
2. Use a sharp pencil to place dots on the page directly under each of the vertices of the plate, and 2 or more (widely separated) dots directly under each of the strings.

Notes:

The brass plate and the strings will be a short distance in front of the vertical force board. To place a dot directly under a string, etc. you will need to view the string along a line at right angles to the force board (and your A3 page), as illustrated in figure B.

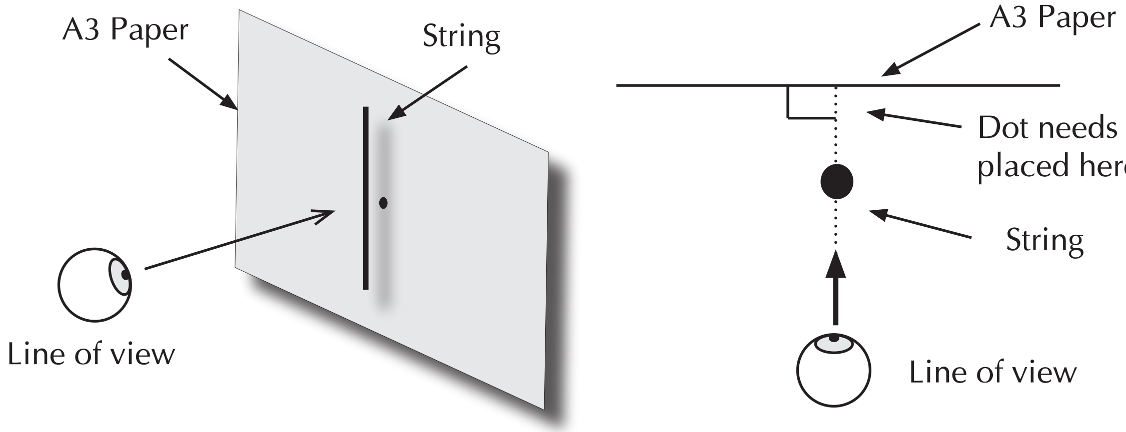


Figure B. Line of sight, side view (left hand side) and top view (right hand side)

· Hold the small mirror flat against the A3 page under a string and look at it (figure C i)
· In general, you will see both the string and its image in the mirror, this image appears to be behind the mirror. (If you stand in front of your bathroom plane mirror you see an image of yourself. Your image is positioned behind the mirror!)

To draw a dot directly behind the string.
· Look along a line at right angles to the board so that the string image is not visible (because it will be covered by the string itself)
· Place dots at either end of the mirror where the string is as shown in figure C ii

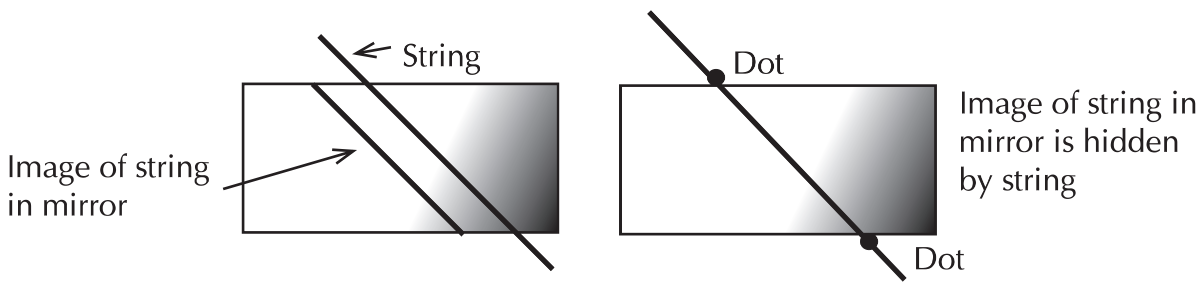


Figure C. The mirror placed behind the string

3. Remove the sheet of paper from the board, and using a ruler draw in an outline of the brass plate, and lines along the directions of each of the strings.

4. Label each of the lines with the magnitude of the total force applied to the plate along that line.

5. Use a protractor to measure the direction of each force

Results, Discussion, and Analysis

6. Summarise your results in the table below and write the units in the second row.

Force name	Mass	Force (magnitude)	Direction
Units			
1			
2			
3			
4			
5			

7. In the space on the next page, construct a scale vector diagram to determine the resultant force acting on the plate. The scale should be chosen so that the diagram will fit but should also be as large as possible so that the resultant force can be read easily.

8. From your scale vector diagram what was the resultant force acting on the plate?

Magnitude: Direction:

Scale vector diagram:

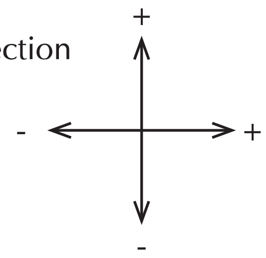
Scale:

Part 3. Calculating the resultant force by resolving the vectors into components

9. On your A3 sheet draw in two perpendicular axes at a convenient orientation. Label one ***axis a*** and the other ***axis b***.
10. For each of the forces acting on the plate determine the magnitude and direction of its components along these two axes, recording the result in the table given. (***All working must be shown***).
11. Determine the resultant force acting on the plate in each of these axis directions.

Force	Component along ***axis a***		Component along ***axis b***	
	Direction* (+/-)	Magnitude	Direction* (+/-)	Magnitude
Resultant				

* By direction we just mean is it pointing in the positive or negative direction



12. Use trigonometry to calculate the final magnitude and direction of the resultant force.

Magnitude:
Direction:

Conclusions

Make some conclusions from your experiment. Make sure you *at least* include discussion of:

- Whether you answered the aim of the experiment
- The major results of the experiment
- A comparison of the results from the two different methods
- Whether your results are consistent with the theory given in the preliminary information
- Possible reasons for discrepancies from theory
- Methods to improve the experiment and/or minimise discrepancies

When you have finished, have a demonstrator check your work.

DEMONSTRATOR USE ONLY

Criteria	***Mark out of***	***Tick/Cross***
Preliminary work and attendance at discussion with demonstrator	2	
The experiment (4 marks)		
Part 1 - Mass of brass plate measured (including units)	1/2	
Part 2 - Accurate replica drawn	1/2	
- Magnitude and direction of each force calculated	1/2	
- All units included	1/2	
- Scale vector diagram drawn	1	
Part 2 - Forces resolved into components	1	
Results, analysis and discussion (4 marks)		
Obtained magnitude and direction of resultant force from part 2	1	
Obtained magnitude and direction of resultant force from part 3	1	
Conclusion	2	

Demonstrator signature: Total mark: /10