Course: **Introduction to Engineering**

Unit: **Problem Solving**

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1. INTRODUCTION

1.1 IMPORTANCE OF PROBLEM SOLVING FOR ENGINEERS

- One of the definitions given for engineering in the first lecture was: *The use of scientific knowledge* ***to solve practical problems****.*
- Problem solving is a fundamental part of engineering.
- Engineers are often employed for their problem solving abilities – in wide ranging occupations.
- Problems in engineering come in many forms and sizes.
- Most problems encountered in tutorial, test and exam papers are "drill exercises", where the plan of attack is based on recent experience and is relatively straightforward.
- Design-style problems may involve many levels of sub-problems, where the plan of attack will need to first identify these sub-problems (**analysis**), before their individual solutions are found and combined into a total solution (**synthesis**).

1.2 SCOPE OF THIS LECTURE

The focus of this lecture is on problem solving, and it provides students with an introduction to:

- A specific problem solving strategy.
- Four types of problem solving tactics, illustrated by means of simple examples.
- Creative thinking.

2. A PROBLEM SOLVING STRATEGY

- A flow diagram for an effective problem solving strategy is shown in Figure 1. (This is just one of many possible strategies.)
- Notice that although the major flow is from *Problem Statemen*t to *Solution*, there are paths that feed back to earlier steps.
- Feedback will occur when it becomes clear that something is missing in the current step.
- In practice, it is unlikely that the solution will be found without the need to re-visit an earlier step, unless the problem is simple or familiar to the solver.
- The important point is to be aware of which step you are in as you work though the solution to the problem.
- Mixing steps is not recommended.
- For example, trying to solve equations while it is unclear if these equations are even relevant is likely lead to a loss of focus and eventual frustration.
- This strategy can be applied to both drill- and design problems. However, it may be an overkill for the more trivial drill problems, and it needs further elaboration for high-level design problems.

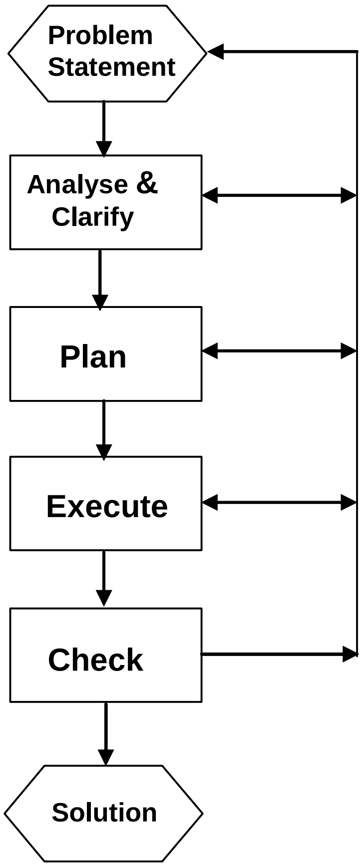

Figure 1: Example of problem solving strategy

2.1 ANALYSE AND CLARIFY THE PROBLEM

- Consider questions such as the following, to analyse and clarify a problem:
  - ∗ What is the purpose of the problem / solution?
  - ∗ Can the problem be interpreted in more than one way?
  - ∗ Is the "real problem" maybe hidden by a "perceived problem" ?
  - ∗ What are the required inputs?
  - ∗ What are the available inputs (known data)?
  - ∗ What are the desired outputs (required results)?
  - ∗ Are there any constraints that must be satisfied?
  - ∗ What assumptions can be made to simplify the task?
  - ∗ How can the validity of the assumptions be tested?
- Drawing diagrams can be very helpful.
- Using thinking tools, such as ***mind maps*** (search the topic with www.google.com to learn more about it) may help to identify all relevant factors in the problem:
  - ∗ Write down a central idea.
  - ∗ Think up new and related ideas, and write it down around the central idea.

  * Look for connections between the ideas.

2.2 PLAN AN APPROACH TO SOLVE THE PROBLEM

- Check for similarities with previous problems.
- If the task is complex, then possible tactics are: (Refer back to *Iteration and the Incremental Approach* in Lecture 1.)
  * break the task down into smaller tasks (**analysis**);
  * solve a related, but simpler task first; and
  * use assumptions and approximations, *but these should be clearly labelled as such*.
- Use general principles (such as standard formulae), but **only if** they are relevant.
- Determine the relevant underlying physical principles that apply to this problem, then assemble equations based on those principles so that the unknowns can be found from a logical sequence of operations.
- Wherever possible, set up a solution strategy that generates meaningful intermediate results while working towards the final result. Intermediate results can give insight into what is going on.
- Use feedback to make sure that the problem definition is still valid, in the light of new information uncovered during the planning and analysis processes.
- Use estimates to find a “ball park” figure for the solution – i.e. *What do I expect the solution to be?* (Refer back to *Gut Feeling and Common Sense* in Lecture 1.)

2.3 EXECUTE THE PLAN

- Work should be clearly laid out so that any errors can be quickly identified and corrected (either by you or by others).
- Systematically combine results of parts of the problem into larger clusters (synthesis), until the complete solution had been compiled.
- Use feedback to make sure that the problem definition and the problem breakdown are still valid, in the light of new information uncovered during the synthesis process.

2.4 CHECK THE RESULTS

- Are the results consistent with expectations from the purpose of the problem, from experience of similar problems, from the ball-park figures, from gut feeling, and from common sense?
- Do the results make physical sense?
- Are the units consistent?
- Are the signs correct?
- Can the results be checked by another process, such as another solution procedure or computer simulation?

- *NOTE: In general, clients do not provide solutions to their problems at the back of the specification sheet. Therefore, being able to check answers at the back of text books is not a useful long-term engineering skill.*
- REFLECTION: Can this solution be used as the basis for solving other related problems?
- Use feedback to make sure that the solution satisfies the original problem statement.

3. PROBLEM SOLVING TACTICS

- Students are encouraged to build up their own list of problem-solving tools based not only on literature searches, but also on their own experiences.
- The list of useful tools will increase as you think about each problem that you solve, and reflect on how you solved it or on why you didn't.

3.1 GENERALIZING

- It is often useful to solve for a general case and then apply the results to a specific case.
- The advantage in this approach is that it is easier to check the validity of the answer, and it may also offer insight into the problem, which may be useful for solving similar problems later on.

- Consider the resistor-divider circuit shown in fig.2. Find the voltage across the 10 ohm resistor.

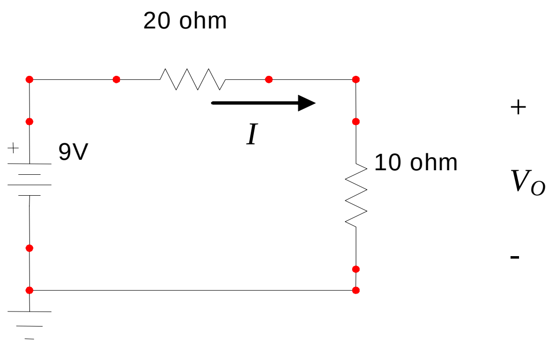

Figure 2

3.1.1 Identify the Physical Principles

<u>Ohm's Law</u>

The voltage drop across a resistor is equal to the resistance value times the current that flows through the resistor, or

$$V = RI \qquad (1)$$

Kirchhoff's Voltage Law (KVL)

The sum of voltage drops around a connected loop of elements is zero (taking care to observe the ***directions*** or ***polarities*** of all voltage drops), or

$$\sum_{loop} V_i \quad = \quad 0 \qquad (2)$$

Kirchhoff's Current Law (KCL)

The sum of currents entering a node (a point where two or more elements are connected together) is zero (taking care to observe the ***directions*** of all current flows), or

$$\sum_{node} I_i \quad = \quad 0 \qquad (3)$$

3.1.2 Sketch the Problem

In this case, the circuit diagram already exists. However, more insight may be gained if the circuit is redrawn as shown in fig.3.

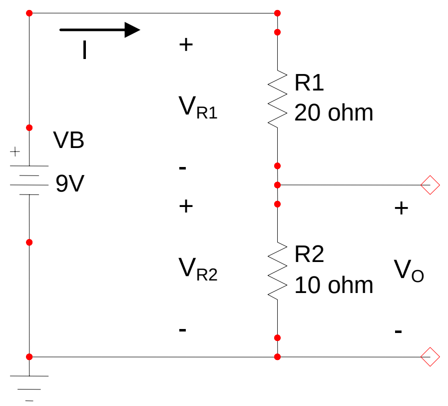

Figure 3

This re-drawn figure is useful for the following reason: the voltage drop from the top to the bottom of the circuit is 9V, irrespective of whether we follow a path through the battery (which gives an obvious 9V drop), or through the two resistors. In other words, this diagram helps us to re-write KVL as

$$9 \quad = \quad V_{R1} + V_{R2} \qquad (4)$$

It should also be clear from this diagram that the same current *I* will flow through both resistors (as there is no closed path that would enable current to flow out of the output terminals):

$$I\left(through\ R_1\right) \quad = \quad I\left(through\ R_2\right) \quad = \quad I \qquad (5)$$

This is a simple application of KCL.

3.1.3 Find the Specific Solution

Using ***Ohm's Law***:

$$V_O = 10\,I \qquad (6)$$

Using ***Kirchoff's Voltage Law*** (***KVL***):

$$9 = 20\,I + 10\,I \qquad (7)$$

$$\begin{aligned} I &= \frac{9}{20 + 10} \\ &= 0.3\,ampere \end{aligned} \qquad (8)$$

Substitute eq.(8) into eq.(6) gives

$$\begin{aligned} V_O &= 10 \, x \, 0.3 \\ &= 3 \, volt \end{aligned}$$

3.1.4 Find the General Solution

Now, use only the labels shown in fig.3.

Using ***Ohm's Law***:

$$V_O = R_2\, I \qquad (9)$$

Using ***Kirchoff's Voltage Law*** (***KVL***):

$$V_B = R_1\, I + R_2\, I \qquad (10)$$

$$\begin{aligned} I &= \frac{V_B}{R_1 + R_2} \\ \therefore \; V_O &= \frac{R_2}{R_1 + R_2} V_B \end{aligned} \qquad (11)$$

Substitute values into eq.(11) gives:

$$\begin{aligned} V_0 &= \{10 / (10 + 20)\}\, 9 \\ &= 3 \, volt \end{aligned}$$

- The general solution takes a little longer, but it gives more ***insight*** into how the circuit works. For example, notice that the voltage V_O depends on the ratio of the two resistor values.

- The value of V_0 will be 3 volt for any resistor values for which R2 / (R1 + R2) = 1/3.
- We can see this more clearly by re-writing the general solution as:

$$V_O = \frac{1}{1 + \left(\frac{R_1}{R_2}\right)} V_B \tag{12}$$

- Eq.(12) is now a general rule that can be applied to similar problems in the future.
- In fact, eq.(12) is so useful that it has two names: the ***voltage-divider*** or ***resistor-divider rule***.

- An analytic expression such as eq.(12) is usually easier to check for correctness than a numerical solution.
- One way to check eq.(12) is to insert extreme element values where the result is known.
- For example, if we set R_2 to zero, then this is equivalent to a ***short circuit***, and so V_O should be zero; and eq.(12) does, in fact, give this result.
- Alternatively, if we set R_2 to infinity, then this is equivalent to an ***open circuit***. In this case no current flows, and so we expect $V_O = V_B$ = 9V. Once again, this is the answer given by eq.(12).
- Although these checks do not guarantee that eq.(12) is correct, they do increase the likelihood that it is correct.

3.2 USING APPROXIMATIONS

Consider the circuit shown in fig.4. For this circuit, what is the value of V_O?

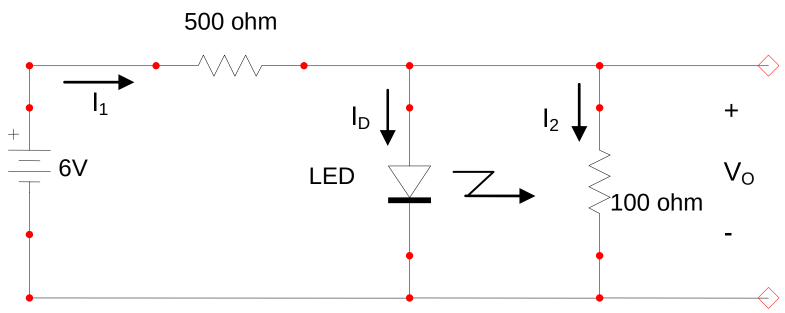

Figure 4

- This circuit is similar to the previous circuit, but includes a light emitting diode (LED) across the output resistor.
- We can solve this problem if we know the relationship between the voltage drop across the diode and the current that passes through this diode.

- This relationship can be derived from the physics of the diode structure:

$$I_D = I_S \left(e^{V_D / nV_T} - 1\right) \qquad (13)$$

- This is a ***nonlinear*** equation that is relatively difficult to use, and leads to a complicated design equation.
- Instead, it is possible to replace this equation by a much simpler equation that gives approximately the same result but with much less effort.
- The simpler approximate model is a ***piecewise linear*** pair of equations:

  if $I_D > 0$ then the diode is ON and $V_D = V_{D,on}$

  else (14)

  if $V_D < V_{D,on}$ then the diode is OFF and $I_D = 0$
- The effect of this approximation is shown in fig.5, which gives current versus voltage plots for eqs.(13) and (14).

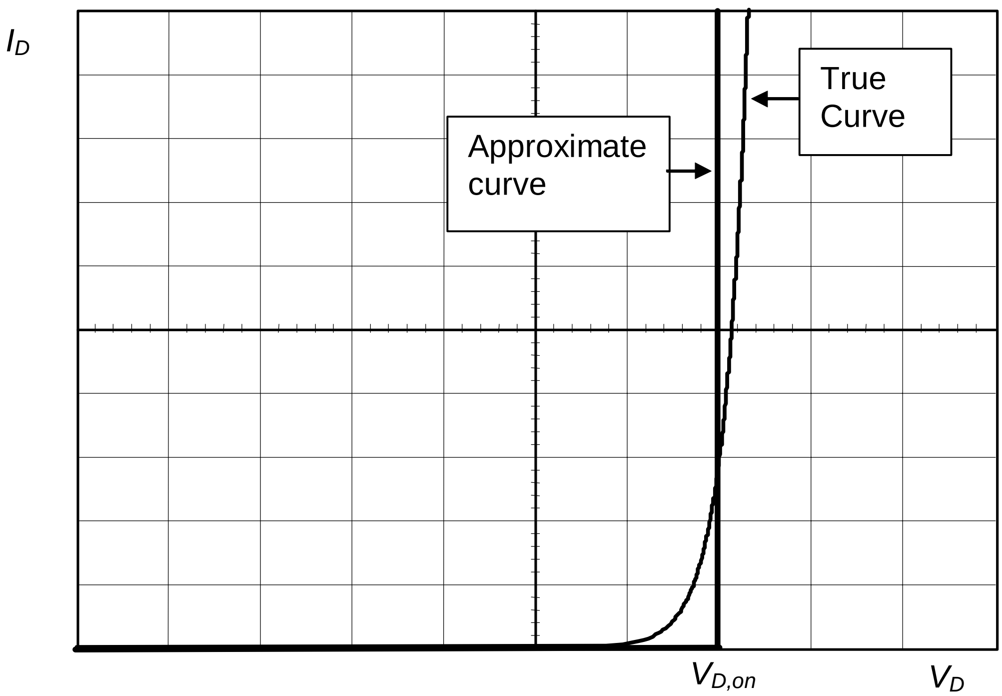

Figure 5

- Usually the value for $V_{D,on}$ for a particular LED is given by the manufacturer in their data sheets for that particular device.
- The manufacturer will also specify the maximum current that can flow through the diode without damaging it.

- For this problem, we will use a LED with $V_{D,on}$ = 2V and I_D(max) = 0.02A.

- Eq.(14) is simple to apply, but it does require us to know whether the diode is ON or OFF.
- If we don't know whether it is or it isn't, then we will have to assume that it is in one of these states, and then test for consistency.
- To begin, we will assume that the LED is ON, in which case

  $V_O \;=\; V_D \;=\; V_{D,on} \;=\; 2V$
- This seems a simple solution, but is it correct? We now need to check the validity of our initial assumption that the diode is ON: our assumption will be valid only if $I_D > 0$.
- Referring to fig.(4), from Ohm's law:

$$I_2 \;=\; \frac{2}{100} \;=\; 0.02\ A$$

- Next, from KVL:

$$6 \;=\; 500\, I_1 \;+\; 2$$
$$\therefore\; I_1 \;=\; \frac{6\;-\;2}{500} \;=\; 0.008\ A$$

- Finally, using KCL:

$$I_1 \;=\; I_D \;+\; I_2$$
$$\therefore\; I_D \;=\; 0.008 \;-\; 0.02 \;=\; -0.012\ A$$

- This value for I_D is inconsistent with our assumption that the diode is ON. Therefore, the diode must, in fact, be OFF.
- So we must repeat the analysis, but this time we use the model for an OFF diode, which is I_D = 0.
- With I_D = 0, the diode plays no part in the operation of the circuit, so it can be removed, as shown in fig.6.

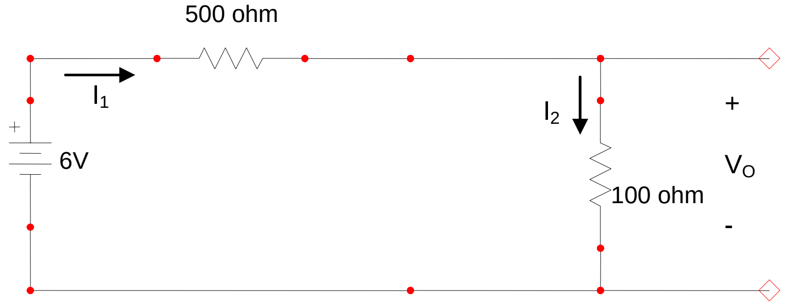

Figure 6

- This circuit is just a simple resistor divider that we considered earlier. Using eq.(12):

$$V_O = \frac{1}{1 + \left(\frac{R_1}{R_2}\right)} V_B$$

$$= \frac{1}{1 + \left(\frac{500}{100}\right)} \times 6 = 1\,volt$$

- Of course, we should now check for consistency of the diode model.
- Now, $V_O = V_D$, so that $V_D <$ 2V, as required. So this time our results are consistent, and the final answer is

  V_O = 1 *volt*

3.3 USING THE ANSWER TO SOLVE THE PROBLEM

- One of the characteristics of a design problem is that the answer is sometimes known when the task is defined – that is, we already know what we want, we just have to find a way to get there!
- Return to LED example: This time, the task is to find a suitable value for R1 such that the diode in ON, and operates at its maximum rated current, which is 20mA.
- The circuit is re-drawn to include the unknown R1 in fig.7.

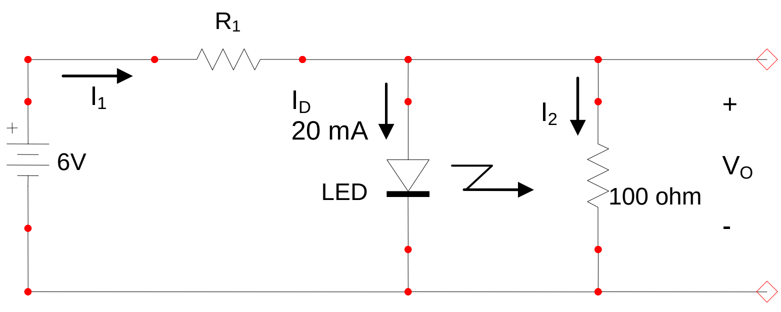

Figure 7

- To begin,

  $V_O = V_{D,on} = 2V$

- In this case, this equation is ***NOT an assumption*** – it is a consequence of the fact that the diode must be turned ON.
- We also know that

  $I_D = 0.02$ A

- Also, from Ohm's Law:

$$I_2 \quad = \quad \frac{V_O}{R_2} \quad = \quad \frac{2}{100} \quad = \quad 0.02\ A$$

- Next, from KCL:

$$\begin{aligned} I_1 \quad &= \quad I_D \ + \ I_2 \\ &= \quad 0.02 \ + \ 0.02 \quad = \quad 0.04\ A \end{aligned}$$

- Finally, from KVL:

$$\begin{aligned} 6 \quad &= \quad R_1\, I_1 \ + \ V_O \\ &= \quad 0.04\, R_1 \ + \ 2 \\ \therefore \quad R_1 \quad &= \quad \frac{6 \ - \ 2}{0.04} \quad = \quad 100\ ohms \end{aligned}$$

- Notice that this design problem was easier to solve than the previous analysis problem because we had an extra fact – the diode is ON – to work with.

3.4 ITERATIVE REFINEMENT

- Sometimes a trial-and-error approach to a problem may suggest a more elegant solution to the problem, which may in turn lead to still more elegant solutions.
- Example (Holtzapple & Reece 2005): "Inscribe a square in any given triangle. Two vertices of the square should be on the base (the longest side) of the triangle, the other two vertices of the square on the other two sides of the triangle, one on each."
- Draw a rough figure:

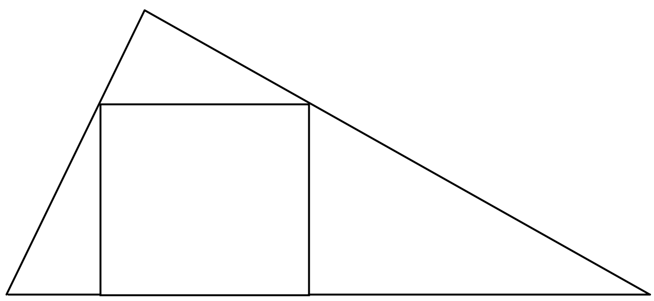

Figure 8

- This problem can be solved, eventually, by ***trial and error***.
- A systematic approach is to start with a small square with one vertex on the left side, and then to progressively increase the size of the square until the remaining vertex touches the right side, as

shown in fig.9.

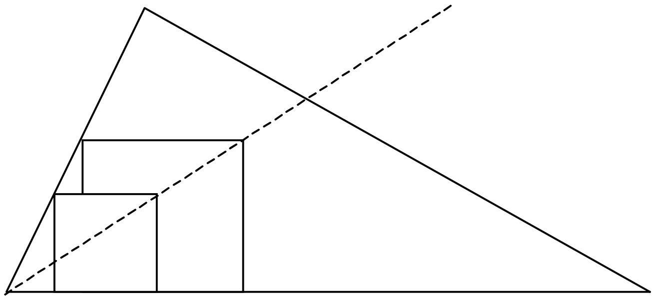

Figure 9

- In following this strategy, we notice that the upper right vertices all lie on a line that starts from the lower left corner of the triangle.
- This leads to a simple construction technique:
  - ∗ First draw one square with its upper left vertex touching the left side of the triangle.
  - ∗ Next, draw a line that passes through the lower left vertex of the triangle and the upper right vertex of the square.
  - ∗ Extend this line until it touches the right side of the triangle.
  - ∗ This point of intersection defines the upper right vertex of the final square, from which all sides of the final square can be easily constructed.
- This works fine, but it leads to the question of why does this work? And, if it does work, is there an easier way to specify the construction of the square?
- We suspect that from the graphical construction, a little bit of geometry might provide an answer to these questions.
- Consider the defining lengths and angles shown in fig.10, which shows the triangle and a square with upper left vertex touching the left side of the triangle.

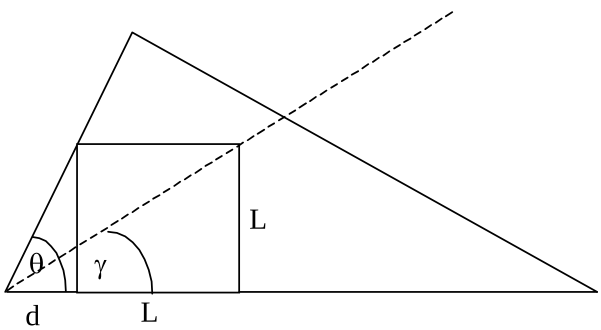

Figure 10

- From geometry, the known angle   is given by:

$$\tan\theta \quad = \quad \frac{L}{d} \tag{15}$$

- while the unknown angle is given by

$$\tan\gamma \quad = \quad \frac{L}{d \;+\; L} \tag{16}$$

- Here we have two equations in three unknowns.
- It may appear that a third equation is needed.
- But, based on the observations that the vertices of the sequence of squares lay along a line (where *L* and *d* will change with each square), we should not need the actual values for *L* and *d.* Only their ratio seems important.
- Re-writing eq.(16) gives

$$\begin{aligned} \tan\gamma \quad &= \quad \frac{\frac{L}{d}}{1 \;+\; \frac{L}{d}} \\ &= \quad \frac{\tan\theta}{1 \;+\; \tan\theta} \end{aligned} \tag{17}$$

- Eq.(17) implies that the revised construction process is to extend a line from one vertex out to the opposite side of the triangle at an angle given by:

$$\gamma \quad = \quad \tan^{-1}\left(\frac{\tan\theta}{1 \;+\; \tan\theta}\right) \tag{18}$$

- Notice that this is the second time that we have met a formula that has been defined by a ratio of unknowns.
- Solving for ratios rather than absolute values will reduce the number of unknowns by 1.
- So this is a useful thing to look out for, and would become a useful addition to a toolbox of problem-solving techniques.

4. CREATIVITY

4.1 THINKING TOOLS

- Creating new solutions to new problems (or even new solutions to old problems) can be a hit-and-miss affair.
- However, the mind that is prepared for the task is more likely to be successful than the mind that is not.

- A number of authors have proposed techniques that increase the chances of success in a creative task.
- These techniques are often called thinking tools.
- Two very popular techniques are:
  - The “Six Thinking Hats” by Edward de Bono; and
  - “Mind Maps” by Tony Buzan.
- The underlying principle for many of these techniques is that they encourage *thinking about thinking*.
- They are useful guides (tools) rather than prescriptive steps that must be followed.
- Often, the process of thinking about a problem is much more important than specific details of the solution.

4.2 BARRIERS TO CREATIVE THINKING

- ***Being overwhelmed by the problem***
  - break the problem down into smaller manageable problems;
  - first solve similar but simpler problems;
  - take a break – our sub-conscious is often better at extracting a solution from a mass of input data that our conscious is – many solutions are found after sleeping on it;
  - developing a broader range of interests can enable an engineer to see a problems from more points of view, and it may be that one of these other points of view leads more directly to the solution to the problem;
  - persistence: one reward for toughing it out is *serendipity*, where a solution to a much bigger problem is suddenly revealed while concentrating on a lesser problem.
- ***Underestimating / allowing insufficient time, space or resources to the task***
- ***Thinking hard instead of thinking smart***
  - Sometimes a little learning can be a dangerous thing
  - Placing too much faith in logic or complicated theories may mean that unusual or simpler alternatives are overlooked.
- ***Misdirection***
  - By using pre-conceived ideas; or
  - by following a biased problem description.
  - *Example (Mack 1992):*
    - Three engineers rented a hotel room that was supposed to cost $40 for the night (a long time ago).
    - The desk clerk mistakenly charged them $15 each, payable in advance.
    - Later he realized he had overcharged them, but couldn’t figure out how to divide the five-dollar refund among the three engineers.
    - So he pocketed two dollars and returned one dollar to each of the engineers.

- The engineers ended up paying $14 each, or $42.
- That, plus the clerk's two dollars, added up to $44.
- What happened to the other dollar?

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6. SELF-ASSESSMENT

6.1 TRUE / FALSE QUESTIONS

Indicate which of the following statements are TRUE and which are FALSE.

1. There is one problem solving strategy suitable for all situations.
2. When solving a problem, checking for similarities with previous problems is cheating, and therefore not allowed or recommended.
3. Asking "*What do I expect the solution to be?*" can be helpful, but is not really necessary to solve problems.
4. Being able to check answers at the back of text books is a useful long-term engineering skill.
5. It is often useful to solve for a general case and then apply the results to a specific case.
6. Finding a general solution might take a little longer, but it usually gives more insight into a problem.
7. One way to check a general solution is to insert extreme values to see whether expected results are obtained.
8. Using approximations is not a valid engineering method.
9. One of the characteristics of a design problem is that the answer is sometimes known when the task is defined.
10. Creating new solutions to new problems (or even new solutions to old problems) can be a hit-and-miss affair.

6.2 MULTIPLE CHOICE QUESTIONS

Choose the one most correct answer for each of the following questions:

1) In the flow diagram used to represent a problem solving in this lecture:
   a) The major flow is from Solution to Problem Statement.
   b) Arrows point in all directions to indicate flexibility in the approach.
   c) Both (a) and (b).
   d) None of the above.
2) When problems are solved:
   a) It is not allowable to make any assumptions.
   b) Assumptions can be made, as long as their validity is tested.
   c) Assumptions always lead to easy, but wrong answers.
   d) Assume the worst case.
3) Drawing a mind map include steps such as:
   a) Write down the solution.
   b) Think up new technologies.
   c) Look for connections between ideas.
   d) All of the above.
4) Generalizing is a valid problem solving tactic, but:
   a) General results are not useful for specific problems.
   b) Checking whether the result is correct is almost impossible.
   c) It is only suitable for circuit analysis.
   d) None of the above.
5) Using approximations are:
   a) The lazy engineer's way of getting something for nothing.
   b) Useful, and often essential in engineering.
   c) Unnecessary with our access to powerful computers.
   d) None of the above.
6) Using the answer to solve the problem is:
   a) Only valid for text book problems.
   b) Cheating.
   c) Sometimes possible when the desired solution is known.
   d) All of the above.
7) Iterative refinement is used when:
   a) Trial and error can give useful pointers in the direction of the solution.
   b) When we don't know what's going on initially.
   c) We seek more elegant solutions.
   d) All of the above.
8) Problem solving tactics is useful for:
   a) Solving simple circuit analysis problems.

b) Geometric problems.
c) All types of problems.
d) None of the above.

9) The process of finding a solution for a problem is:
a) A waste of time. Just solve the problem!
b) Often much more important than specific details of the solution.
c) A nice theoretical concept, with limited practical use.
d) All of the above.

10) A major barrier to creative thinking is:
a) Being unimpressed by a problem.
b) Allowing too much time for the task.
c) Thinking hard instead of thinking smart.
d) All of the above.

6.3 SHORT ESSAY QUESTIONS / EXERCISES

Write a short essay (with at least 10 facts) on one of the following topics. Do not merely copy the course notes, but write the essay **in your own words** such that your understanding of the topic becomes clear.

1. Creative thinking.
2. Problem solving tactics.
3. A problem solving strategy.
4. The validity of approximations in engineering problem solving.

Exercise: If the load resistor in figure 7 (that is, the resistor in parallel with the LED) is removed, then find the new current that passes through the LED. Will the LED survive the experience?

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