

# A Unifying Model for the Structure of Intellectual Abilities

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Models of the structure of cognitive abilities suggested by Spearman, Thurstone, Guilford, Vernon and Cattell-Horn are reviewed. It is noted that some of the models include a general intellectual factor ( $g$ ) while others do not. It is also noted that some models are nonhierarchical, while in others more narrow abilities are subsumed under broader abilities in a hierarchical pattern. An empirical study in which a test battery of 16 tests was administered to some 1000 subjects in the 6th grade is reported. Using the LISREL technique to test different models, good support is obtained for oblique primary factors in the Thurstone tradition as well as for the second-order factors fluid intelligence, crystallized intelligence, and general visualization hypothesized by Cattell and Horn. It is also found, however, that the second-order factor of fluid intelligence is identical with a third-order  $g$ -factor. On the basis of these results a three-level model (the HILI-model) is suggested, with the  $g$ -factor at the top, two broad factors reflecting the ability to deal with verbal and figural information, respectively, at the second-order level, and the primary factors in the Thurstone and Guilford tradition at the lowest level. It is argued that most previously suggested models are special cases of the HILI-model.

Models for the structure of abilities are legion, and names such as Burt, Cattell, Guilford, Guttman, Horn, Jensen, Spearman, Thomson, Thurstone and Vernon have come to be associated with different models. While all the models are based upon results from multivariate statistical analysis, most of them have a close relationship with a particular kind of factor analysis.

Among the models based upon factor analysis, one line of demarcation goes between models which postulate a general factor of intelligence (e.g., Burt, 1949; Spearman, 1904b; Vernon, 1950), and models which do not allow for a general intellectual factor (e.g., Cattell, 1971; Guilford, 1967; Horn & Cattell, 1966; Thurstone, 1938). Another line of demarcation goes between hierarchical models (e.g., Burt, 1949; Horn & Cattell, 1966; Vernon, 1950) and models which treat all the dimensions as being of equal generality (e.g., Guilford, 1967; Thurstone, 1938).

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It has proven difficult to secure conclusive evidence in favor of one of the models. Thus, Sternberg (1981a) asked: "Almost eighty years after the first presentation of Spearman's (1904) two-factor theory, has anyone answered through factorial means the question of whether a general factor exists?" (Sternberg, 1981a, p. 143). The answer given was in the negative. Sternberg argued, further, that factor analysis has "... failed because it has been too successful in supporting, or at least in failing to disconfirm, too many alternative models of intelligence." (Sternberg, 1981a, p. 143).

Opinions strongly differ as to which techniques of factoring and rotation are preferable and choice of technique has largely been made on subjective grounds. Since the characteristics of the different models of intelligence to a large extent derive from the factor analytic technique relied upon, it may be argued that the "success" of factor analysis in supporting many alternative models of intelligence is above all a methodological problem.

However, with the advent of confirmatory factor analysis (e.g., Jöreskog, 1969; Jöreskog & Sörbom, 1978) a sharper factor analytic tool has become available for investigating models of the structure of ability. The purpose of the present paper is to take advantage of this new development in the study of the organization of cognitive abilities. First, however, some of the most important models of the structure of abilities will be reviewed in greater detail.

## PREVIOUS MODELS

The major models of ability are so well known that no closer presentation is necessary in this context. For purposes of common reference, however, their most important characteristics will be briefly mentioned.

### *The Spearman Model*

Spearman (1904a) developed the Tetrad Difference method to investigate whether one common factor is sufficient to account for the intercorrelations among a set of tasks. Applying this technique to several different sets of variables, comprising among other things psychophysical assessments, ratings of intelligence, school marks, and psychological tests, he was able to conclude (Spearman, 1904b, 1927) that in most cases one underlying factor accounted for the intercorrelations. On the basis of these results he proposed the Two Factor theory of intelligence, which states that performance on an intellectual task is affected by two factors only: one general, common to all tasks (*g*), and one which is specific to the task (*s*).

### *The Thurstone Model*

During the 1920s and 1930s more refined factor-analytic techniques were developed. As is well known, the major contributor was Thurstone (1931, 1938,

1947), who developed factor analysis to encompass multiple common factors. Thurstone also introduced the principle of "simple structure" which essentially states that any test should be affected by one or a few factors only. Applying the new techniques to large test batteries (e.g., Thurstone, 1938), about a dozen factors could be identified, each of which accounted for performance on a subset of the tests in the battery. There was no sign of a general factor.

Most factors identified by Thurstone (1938) were replicated (e.g., Thurstone, 1940; Thurstone & Thurstone, 1941), and it was possible to set up a list of six or seven Primary Mental Abilities (PMA's), such as Verbal Comprehension (V), Word Fluency (W), Induction (I), Space (S), and Perceptual Speed (P).

In the first set of analyses the PMA's were kept orthogonal. But when test batteries were assembled to measure the factors, it was found that the tests were intercorrelated, which led Thurstone to adopt instead an oblique factor model.

The techniques of factor extraction and rotation invented by Thurstone have by now been considerably refined (Harman, 1967). The basic principles remain the same, however, and the Thurstonian Multiple Factor analysis has evolved into the dominating factor analytic technique.

The basic PMA model, with a handful of factors, has been elaborated upon too. By investigation of new domains of ability, and by showing that several of the original PMA's are differentiable into more narrow factors, the list of factors has been extended considerably. Reviews of the research (e.g., French, 1951; French, Ekstrom, & Price, 1963; Horn, 1977; Pawlik, 1966) indicate that it may be necessary to distinguish as many as 30 to 50 factors of ability. While some of these factors are broad and comprehensive, others are very narrow and circumscribed, and must be considered the result of a subdivision of a broader factor. However, all factors are primary in the sense that they represent results from applications of Multiple Factor analysis to matrices of intercorrelations between tests.

### *The Guilford Model*

The "Structure-of-Intellect" (SI) model (e.g., Guilford, 1967) may be seen as an attempt to bring order to the multitude of factors. This model describes tests and factors in terms of the three facets operation, content and product. Combination of the facets yields 120 possible factors, and in the latest version of the model Guilford and Hoepfner (1971) claim identification of at least one factor (and sometimes more) in each of 98 of the cells in the model.

### *The Burt and Vernon Models*

In the British research on abilities Multiple Factor analysis has had less impact, and the *g* factor did not vanish altogether. In the post-Spearman work it was soon to be discovered, however, that in addition to *g* there are also group-factors of great importance. Factor analytic techniques were developed, which

from a matrix of correlations extract first the *g* factor, and then group-factors of successively smaller breadth. These hierarchical group-factor techniques (e.g., Burt, 1941; Harman, 1967) thus supply information both about a general factor and about group-factors.

One hierarchical model, summarizing the results of very many analyses, was proposed by Burt (1949). This model seems, however, to have been too much of a logically constructed classification scheme to earn any great impact. Instead, a rather similar model, presented only slightly later by Vernon (1950, 1965) has received more widespread attention.

At the top of the Vernon model there is the *g* factor, and at the next level below there are two major group-factors: verbal-educational (*v:ed*) and spatial-practical-mechanical (*k:m*) ability. The *v:ed* factor subdivides into different scholastic factors, such as number factors and linguistic abilities. The *k:m* factor subdivides too and this complex includes minor group-factors such as perceptual, spatial and mechanical factors. Each of these minor factors can then be subdivided by more detailed testing.

### *The Cattell-Horn Model*

Another hierarchical model has been proposed by Cattell and Horn. The basic concepts of this model were developed by Cattell (e.g., 1940), but the model was neither elaborated upon, nor put to empirical tests until considerably later (e.g., Cattell, 1963; Horn, 1968; Horn & Cattell, 1966).

Methodologically the Cattell-Horn model is based upon oblique Multiple Factor analysis of several orders. Thus, in the first step an ordinary oblique Multiple Factor analysis is conducted, typically yielding a large set of primary or first-order factors. The correlations between the primary factors are then subjected to another factor-analysis which yields secondary or second-order factors. In principle, this procedure of factoring may be continued at successively higher orders, but Cattell and Horn have chosen to stop the factoring at the second-order level.

In the Horn and Cattell (1966) formulation the model includes 5 second-order or "general" factors. The two most important ones are fluid intelligence (*Gf*) and crystallized intelligence (*Gc*). Both are viewed as aspects of general intelligence and are said to involve abstraction, concept formation, and perception and education of relations. *Gf*, however, is involved in tasks which are new to the examinee, while *Gc* is shown in tasks with verbal-conceptual content. *Gf* is thought to represent influences of biological factors and incidental learning on intellectual development, while *Gc* is interpreted as reflecting education and experience. Primary factors such as Induction (*I*) and Cognition of Figural Relations (*CFR*) involve *Gf*, while *Gc* is involved in primaries such as Verbal Comprehension (*V*) and Cognition of Semantic Relations (*CMR*).

The other three second-order factors in the Cattell-Horn model are General

visualization (Gv), General fluency (F or Gr), and General speediness (Gs). Gv appears in almost all tasks with figural content and runs strongly through primaries such as Visualization (Vz), Flexibility of Closure (Cf), Speed of Closure (Cs) and Spatial Orientation (S). Gr reflects the flexibility with which labels for cultural concepts are recalled and recognized, and is involved in the primary factors Associational fluency (Fa), Word Fluency (Fw), and Ideational Fluency (Fi). Gs is defined as quickness of performance, and shows up most clearly in primary factors defined by very simple tasks, such as Perceptual Speed (P) and Numerical Facility (N).

The model has been subjected to empirical tests in large scale factor analytic studies by among others Cattell (1963), Horn (1972), Horn and Cattell (1966) and Undheim (1976, 1978, 1981a). These studies have in general confirmed the hypothesized structure.

It should also be pointed out that the basic model as described before has been elaborated upon by Cattell (1971), Horn (1982) and Horn and Stankov (1982). It would seem, however, that the model in the 1966 formulation so far has had greatest impact and has obtained the best empirical support.

### *Discussion*

Among the models mentioned above, those based upon Multiple Factor analysis have dominated differential psychology during the last three or four decades. One reason for this is that they do show a good fit to empirical data; another reason may be the ready availability of computer programs to perform such analyses.

Multiple Factor analysis was introduced because of an obvious need to differentiate between more factors than was allowed by the Spearman technique. However, by now so many factors have been identified that the results appear almost incomprehensible and of limited practical utility (e.g., Cronbach & Snow, 1977; pp. 152–164; Humphreys, 1962; McNemar, 1964). Undheim (1981c) even went so far as to argue that:

The widespread application of multiple factor analysis in research on abilities seems to have carried factor analysis far beyond its descriptive and conceptual limitations as a research tool, resulting in an ever increasing number of factors of "the mind". It is somewhat paradoxical that whereas the multiple-factor model developed by Guilford . . . tried explicitly to save ability research from this empirical chaos, it, in fact, may have contributed to further "empiricism" in focusing on filling the empty cells in the box model of 120 factors. (Undheim, 1981c, p. 22)

The problems associated with the multitude of factors are severed by the fact that Multiple Factor analysis, in a technical sense at least, considers all factors as being of equal importance. In the hierarchical models, in contrast, the lower-

order factors are subsumed under higher-order factors, which makes these models parsimonious, while they at the same time provide the richness of description of the Multiple Factor models. Thus, in the Cattell-Horn model the Thurstonian primaries, along with several of the factors in the SI-model, are found at the first-order level. This model, therefore, is compatible with the Multiple Factor models, but it obviously goes much beyond these.

During the last few years the hierarchical type of model has gained increasing attention (e.g., Cronbach & Snow, 1977; Snow, 1977, 1980). Given the advantages of this type of model it may be asked, however, why the influence has not been even greater. The most important reason for this is probably the rather great technical difficulties involved. Another reason may be the fact that there are several competing models, which seem so different that if one of them is correct, the others must necessarily be refuted.

The two most important hierarchical models are those contributed by Cattell-Horn and Vernon. Cattell (1963) and Horn (1968), in particular, have claimed the superiority of their model. They argue that Vernon's *v:ed* should correspond to their *Gc*, and that *k:m* corresponds to a mixture of *Gf* and *Gv*.

In their view, then, a major difference between the models is that where Vernon distinguishes two factors, they distinguish three. A similar point was made by Humphreys (1967), who argued that the answer to the question whether *Gf* and *Gv* are distinct factors determines whether the Cattell-Horn or the Vernon model should be accepted.

However, another set of relationships between the models may also be hypothesized. The kind of tests identified to measure *Gf* comes very close to the kind of tests that Vernon lists as measures of *g* in his model. It may, therefore, be argued that *Gf* in the Cattell-Horn model is more or less the same factor as the *g* factor in the Vernon model (cf. Vernon, 1969, p. 25). If this is true, it also would seem natural to equate *Gv* with *k:m*, and *Gc* with *v:ed*. Thus, if a third level, representing the *g* factor, is added to the Cattell-Horn model, the most essential point of difference between the two major hierarchical models would be resolved. Such a combined hierarchical model would, furthermore, be compatible with most of the nonhierarchical models already described. It is, of course, an empirical question whether the second-order factor labeled *Gf* by Cattell and Horn is indeed the same factor as the one called *g* by Spearman and Vernon, and it is also an empirical question whether the other parts of this hypothesized model fit empirical data. It may be noted, however, that Undheim (1981b, 1981c), in particular, has gone far towards a "restoration of general intelligence" on the basis of a reinterpretation of *Gf* in terms of *g*.

The factor analytic techniques employed in previous research tend to be biased in favor of one of the models. These techniques also are fraught with the problem that they are exploratory only, and do not give provisions for statistically sound tests of the number of common factors, or of the significance of factor loadings. Recently, however, factor analytic methods have been devel-

oped which do allow the testing of hypotheses, and which are flexible enough to allow an almost infinite range of different models to be specified.

Jöreskog (1969) presented a method for estimating and testing confirmatory factor models, using maximum likelihood methods. In such models the number of factors, and the pattern of loadings is specified in advance, on the basis of whatever previous knowledge is available about the variables being measured. Estimates of parameters in confirmatory models are unique, so the problem of rotation is avoided altogether. Statistical tests also are available with which the fit of the data to the model may be determined.

Jöreskog (1970) generalized the simple confirmatory factor analytic model to allow formulation of higher-order models, and in still further developments a model has been arrived at which, loosely stated, combines the factor analytic methods with path-analytic techniques (linear structural relations, LISREL; Jöreskog, 1973; Jöreskog & Sörbom, 1978, 1981). This latter model is a completely general model which contains all the earlier models as special cases. It would thus seem that major progress has been made in estimating and testing hierarchical models. This technology has been put to use in an empirical study.

## METHOD

The design of the study involved administration of a test-battery to a rather large sample of subjects. The test battery was assembled in such a way that enough primary factors would be represented to make possible identification of the second-order factors Gv, Gf and Gc.

### *The Test Battery*

The test-battery consists of two parts: one with 13 tests of ability, and one with 3 standardized achievement tests. The following tests of ability were included in the battery:

1. *Number Series II.* In the items in this test a series of 5 or 6 numbers are given, and the task is to add two more numbers to the series. Tests of this type have been shown to load the primary factor Induction (I), which in turn loads Gf.
2. *Letter Grouping II.* The items in this test consist of groups of letters, and the task is to decide which group of letters does not belong with the others. This kind of test, too, has been shown to load the I-factor.
3. *The Raven Progressive Matrices.* The items in the Raven test present a matrix of figures in which the figures change from left to right according to one principle, and from top to bottom according to another principle. One figure is missing, however, and the task is to identify this figure. It is not entirely clear to which primary factor the Raven test should be classi-

fied. French, Ekstrom, and Price (1963) would assign this test to the I-factor. Their I-factor is quite broad, however, and it has more the character of a second-order factor than a primary factor. Horn and Cattell (1966) use the Guilford notation (Cognition of Figural Relations, CFR) to classify this test and the same notation is used here. CFR is hypothesized to load GF.

4. *Auditory Number Span*. This is a conventional digit-span test, with digits in series of varying length being read for immediate reproduction. The test may be hypothesized to load the Memory Span (Ms) primary, which factor Horn and Cattell (1966) hypothesize to be weakly related to Gf.
5. *Auditory Letter Span*. This test is identical with the preceding test, except that letters are used instead of digits.
6. *Metal Folding*. In this test the task is to find the three-dimensional object which corresponds to a two-dimensional drawing. Metal Folding may be hypothesized to load the Visualization (Vz) primary, which in turn belongs with Gv.
7. *Group Embedded Figures*. This test consists of items in which the task is to find a simple figure within a more complex figure. The test has been shown to represent the Flexibility of Closure (Cf) factor, which in turn loads Gv.
8. *Hidden Patterns*. Each item consists of a geometrical pattern, in some of which a simpler configuration is embedded, and the task is to identify those patterns which contain the simple configuration. The test is similar to the Group Embedded Figures test and may be hypothesized to load the Cf-factor.
9. *Copying*. In this test each item consists of a given geometrical figure, which is to be copied onto a square matrix of dots. French et al. (1963) classify this test with the Cf-factor.
10. *Card Rotations*. Each item in this test gives a drawing of a card cut into an irregular shape, and the task is to decide whether other drawings of the card are merely rotated, or turned over onto the other side. This test is highly similar to the Thurstone tests Cards, Figures and Flags, which have been shown to define the Spatial Orientation (S) primary. This primary loads Gv.
11. *Disguised Words*. In this test words are presented with parts of each letter missing, and the task is to identify the word. The test is highly similar to the Thurstone (1944) test Multilined Words, which by him was found to load the Speed of Closure (Cs) factor. French et al. (1963) mention, however, that tests like these may also have a loading on a Verbal Closure factor. According to Horn and Cattell (1966) Cs loads Gv.
12. *Disguised Pictures*. In the items of this test drawings are presented which are composed of black blotches representing parts of the object being portrayed, and the task is to identify the object. Tests similar to this one have been found to load the Cs-factor.



13. *Opposites*. In each of the items in this test the task is to select the word which is the antonym of a given word. The test may be hypothesized to load the Verbal Comprehension (V) primary, which in turn loads Gc.

These 13 tests comprise the tests of cognitive ability in the test battery and they were administered on one occasion. In addition, scores are available on Standardized Achievement test in Swedish, mathematics and English. These tests are administered by the class teachers to most pupils in the 6th grade in Sweden.

The Standardized Achievement test in Swedish (SA) consists of six subtests: Spelling; Reading Comprehension; Vocabulary; Word List, which tests the ability to use a word list to find the meaning, spelling and flexion of words; and Sentence Construction which measures punctuation skills.

The Standardized Achievement test in Mathematics (MA) is composed of 5 sub-tests: Numerical Calculations which tests understanding of the number line, the ability to carry out additions, subtractions, multiplications, divisions and calculations with fractions; Per Cent Calculations; Estimates, which tests the ability to make rapid estimates of the approximate results of an expression; Geometry and Diagrams, which measures simple geometrical skills and the understanding of information presented in graphs and tables; and Applied Computations which presents verbally stated problems, most of which require a mixture of the rules of arithmetic for solution.

In the Standardized Achievement test in English (EA) there are 4 subtests: Vocabulary; Listening Comprehension; Forms and Structures, which tests the knowledge of grammatical rules, such as the do-construction and flexion of verbs; and Reading Comprehension.

### *Subjects*

The study comprised 50 classes in the 6th grade (i.e., the pupils are in their 12th year), in two different communities in Sweden. In all 1,254 pupils attended these classes, but for different reasons the battery of cognitive tests could not be administered to 30 pupils. The final sample thus comprised 1224 subjects (602 boys and 622 girls), with an attrition of 2.4%. For the Standardized Achievement tests attrition was greater, however, which is because it is up to the class-teacher to decide whether all, some or none of these shall be administered. Of the sample of 1224 subjects, 981 (or 80.1%) had results on all the Standardized Achievement tests while 113 (9.2%) had not taken any of these. For the remaining 130 subjects scores were available on one or two of the Standardized Achievement tests. All analyses presented in this paper have been performed on the 981 subjects with complete data.

### *Procedures Used in the Testing*

All 13 tests were given at one single occasion for each class. It took about five 40-minute lessons to administer the tests. The class had recesses and lunchbreak

in a regular manner. In most classes three lessons were used before lunch and two after the lunchbreak. Two administrators, one male and one female, gave the tests to 25 classes each. The class teacher was generally not present in the classroom during the testing.

## RESULTS

Analyses of the tests indicated that they all had satisfactory reliabilities, ranging between .67 for Disguised Pictures and .87 for the Raven Progressive Matrices test (for a detailed description of these analyses, see Gustafsson, Lindström, & Björck-Åkesson, 1981).

The conformatory factor analyses were performed in several steps, starting with a model involving first-order factors only, and ending with a model with factors at three levels. In the initial step at each level a theoretically derived model was fitted. If the fit of this model was poor, as evaluated with, among other things, a  $\chi^2$  goodness-of-fit test, it was modified to achieve a better fit before proceeding to the next higher level. In these modifications models were frequently fitted for subsets of the variables, which submodels were then pieced together into one model.

The successive modifications made to the model imply that the analysis is not confirmatory in the strict statistical meaning of the term; it should rather be viewed as exploratory analysis with confirmatory means. As a consequence the  $\chi^2$  test is, of course, not entirely trustworthy. In evaluations of model fit other factors, such as residuals and parameter estimates, were therefore allowed to influence the decision whether to accept a model or to modify it. Another factor making the  $\chi^2$  test less than perfectly suited for judgements of model fit is that the sample size is so large that even trivial deviations from the hypothesized model cause the test-statistic to be significant.

The rather involved nature of each analysis and the many steps needed to arrive at the final model make reporting of results difficult, and in the present context it is impossible to present anything but the initial and the final model at each level. For a slightly more detailed presentation of the results, the reader is referred to Gustafsson et al. (1981).

Analyses were performed with the LISREL IV (Jöreskog & Sörbom, 1978) and the LISREL V (Jöreskog & Sörbom, 1981) programs. For simplicity, some results will be reported in the parlance of these computer programs.

In order to secure a sufficient number of first-order factors, some of the tests were divided into half-tests, which two halves were taken to identify a primary factor. This practice is, of course, not entirely satisfactory since it does assume the test specificity variance to be zero. In most cases, however, this source of variance is likely to be so small that no important bias in the definition of higher-order factors is introduced.

Table 1 shows the pattern of hypothesized loadings of the tests in the primary

TABLE 1  
Hypothesized Pattern of Loadings of Tests in Primary Factors

Test Name	Abbreviation	First-Order Factor
Metal Folding (odd items)	MF-O	Vz
Metal Folding (even items)	MF-E	Vz
Card Rotation (part I)	CR-I	S
Card Rotation (part II)	CR-II	S
Group Embedded Figures Test	GEFT	Cf
Hidden Patterns	HP	Cf
Copying	CO	Cf
Disguised Words	DW	Cs
Disguised Pictures	DP	Cs
Raven (odd items)	RA-O	CFR
Raven (even items)	RA-E	CFR
Number Series II	NS	I
Letter Grouping II	LG	I
Auditory Number Span	ANS	Ms
Auditory Letter Span	ALS	Ms
Opposites (odd items)	OP-O	V
Opposites (even items)	OP-E	V
Swedish Achievement	Sw Ach	Ach
English Achievement	Eng Ach	Ach
Mathematics Achievement	Ma Ach	Ach

factors. The initial model for the first-order level was hypothesized to involve 9 primary factors, with a pattern of loading of tests in factors as is specified in Table 1. Otherwise, the initial model was an ordinary oblique factor model: no constraints were imposed on the correlations between factors, and the errors in the observed variables were assumed to be uncorrelated.

The model was estimated from the matrix of correlations between tests which is presented in Table 2.

The goodness-of-fit test for this initial model was highly significant ( $\chi^2 = 483.23$ ,  $df = 134$ ,  $p < .000$ ), implying that the model must be rejected. Investigating the model it was found that the hypothesized Ach factor was the major source of misfit, and it proved possible to split this factor into one Verbal Achievement factor (Ve Ach), loaded by the English and Swedish Achievement tests, and one Numerical Achievement factor (Num Ach), loaded by Mathematics Achievement and Number Series. Some additional factor loadings were also found, along with correlated errors of measurement for some of the spatial tests. After these modifications the test statistic showed a borderline significance ( $\chi^2 = 166.50$ ,  $df = 117$ ,  $p < .002$ ).

In the second-order model latent variables are introduced to account for the intercorrelations among the primary factors. The initial second-order model was

TABLE 2  
Correlations Between the Tests in the Reference Battery ( $N = 981$ )

	MF-O	MF-E	CR-1	CR-2	GEFT	HP	CO	DW	DP	RA-O	RA-E	NS	LG
MF-O	1.00												
MF-E	.79	1.00											
CR-1	.33	.34	1.00										
CR-2	.40	.41	.74	1.00									
GEFT	.47	.49	.33	.36	1.00								
HP	.42	.43	.44	.45	.48	1.00							
CO	.43	.43	.45	.47	.50	.56	1.00						
DW	.18	.21	.14	.18	.25	.27	.23	1.00					
DP	.26	.24	.14	.18	.22	.20	.20	.38	1.00				
RA-O	.38	.39	.25	.31	.32	.34	.28	.18	.21	1.00			
RA-E	.40	.41	.27	.33	.35	.38	.31	.18	.21	.81	1.00		
NS	.35	.38	.32	.38	.40	.40	.40	.25	.13	.40	.43	1.00	
LG	.35	.34	.35	.38	.41	.41	.39	.22	.15	.40	.43	.52	1.00
ANS	.14	.12	.13	.14	.14	.17	.17	.15	.10	.12	.12	.22	.20
ALS	.14	.13	.19	.15	.17	.21	.19	.16	.08	.17	.17	.27	.25
Op-O	.32	.29	.20	.23	.31	.31	.29	.17	.13	.30	.30	.42	.39
Op-E	.26	.27	.25	.25	.31	.27	.26	.17	.13	.30	.30	.42	.38
Sw	.33	.33	.27	.31	.39	.38	.36	.27	.15	.40	.42	.56	.51
Ma	.38	.42	.34	.40	.43	.41	.41	.21	.11	.41	.43	.68	.49
Eng	.33	.29	.24	.26	.41	.37	.36	.22	.18	.36	.38	.49	.44
	ANS	ALS	Op-O	Op-E	Sw	Ma	Eng						
ANS	1.00												
ALS	.49	1.00											
Op-O	.16	.18	1.00										
Op-E	.14	.19	.70	1.00									
Sw	.21	.32	.68	.65	1.00								
Ma	.18	.23	.50	.50	.66	1.00							
Eng	.20	.29	.56	.56	.78	.61	1.00						

identical with the final first-order model, except that the primary factors were hypothesized to load the second-order factors as specified in Table 3, and, of course, that there were no correlations among primary factors.

This model resulted in a significant test-statistic ( $\chi^2=277.22$ ,  $df=149$ ,  $p<.000$ ). It is possible, however, to form a specific test of the hypothesis that the intercorrelations among the primary factors may be accounted for by the three second-order factors by taking the difference between this test-statistic and the test-statistic for the final first-order model. This test, too, is highly significant ( $\chi^2=110.72$ ,  $df=32$ ,  $p<.000$ ).

In modifications of the second-order model it was found that Num Ach loaded Gf as well as Gc and that there were minor covariances among the residuals of some primaries (Vz, CFR; Cf, Cs; Ms, Ve Ach; and Cf, Ve Ach). After these relaxations of the model the partial test of the second-order model was not

TABLE 3  
Hypothesized Pattern of Loadings of First-Order Factors  
in Second-Order Factors

First-order Factor	Second-order Factor
Visualization (Vz)	Gv
Spatial orientation (S)	Gv
Flexibility of Closure (Cf)	Gv
Speed of Closure (Cs)	Gv
Cognition of Figural Relations (CFR)	Gf
Induction (I)	Gf
Memory Span (Ms)	Gf
Vocabulary (V)	Gf
Verbal Achievement (Ve Ach)	Gc
Mathematics Achievement (Ma Ach)	Gc

significant ( $\chi^2=18.85$ ;  $df=27$ ,  $p < .88$ ), and the overall test of model fit showed a borderline significance only ( $\chi^2=185.35$ ,  $df=144$ ,  $p < .011$ ).

Even though some modifications had to be made of the originally hypothesized second-order model, the essential characteristics of the initial model undoubtedly remain. Since the hypothesized second-order model was derived from the Cattell-Horn model, the results, so far, may be interpreted as supporting this hierarchical model. In the final step of the analysis, a third-order  $g$  factor was introduced to account for the intercorrelations among the three second-order factors.

With three second-order factors a model with one third-order factor is "just identified", i.e., it has the same chi-square and the same degree of freedom as the second-order model. By imposing further restrictions it is, however, possible to define a fully identified model. One such restriction is, of course, given by the hypothesis of primary interest here, namely that  $Gf$  is identical with  $g$ . This hypothesis implies that the residual variance in the second-order  $Gf$ -factor is zero. Estimating the model with this constraint imposed the test of fit yielded a  $\chi^2$  of 188.58 with 145 degrees of freedom. The difference between this statistic and the statistic for the model without a third-order  $g$ -factor is not significant ( $\chi^2=3.23$ ,  $df=1$ ,  $p < .07$ ), which result strongly supports the hypothesis that  $Gf$  is identical with  $g$ .

An alternative explanation may of course be that the test lacks statistical power. This alternative hypothesis can be ruled out, however, since the estimate of the loading of  $Gf$  in  $g$  was very close to unity, and since a very poor fit was obtained when the restriction was imposed for  $Gv$  ( $\chi^2=98.2$ ,  $df=1$ ,  $p < .000$ ) and  $Gc$  ( $\chi^2=126.6$ ,  $df=1$ ,  $p < .000$ ).

The final LISREL model is shown in Fig. 1. In the figure the tests are shown enclosed in squares while the factors are enclosed in circles. Straight arrows indicate direction of influence, and the estimates of these parameters may be

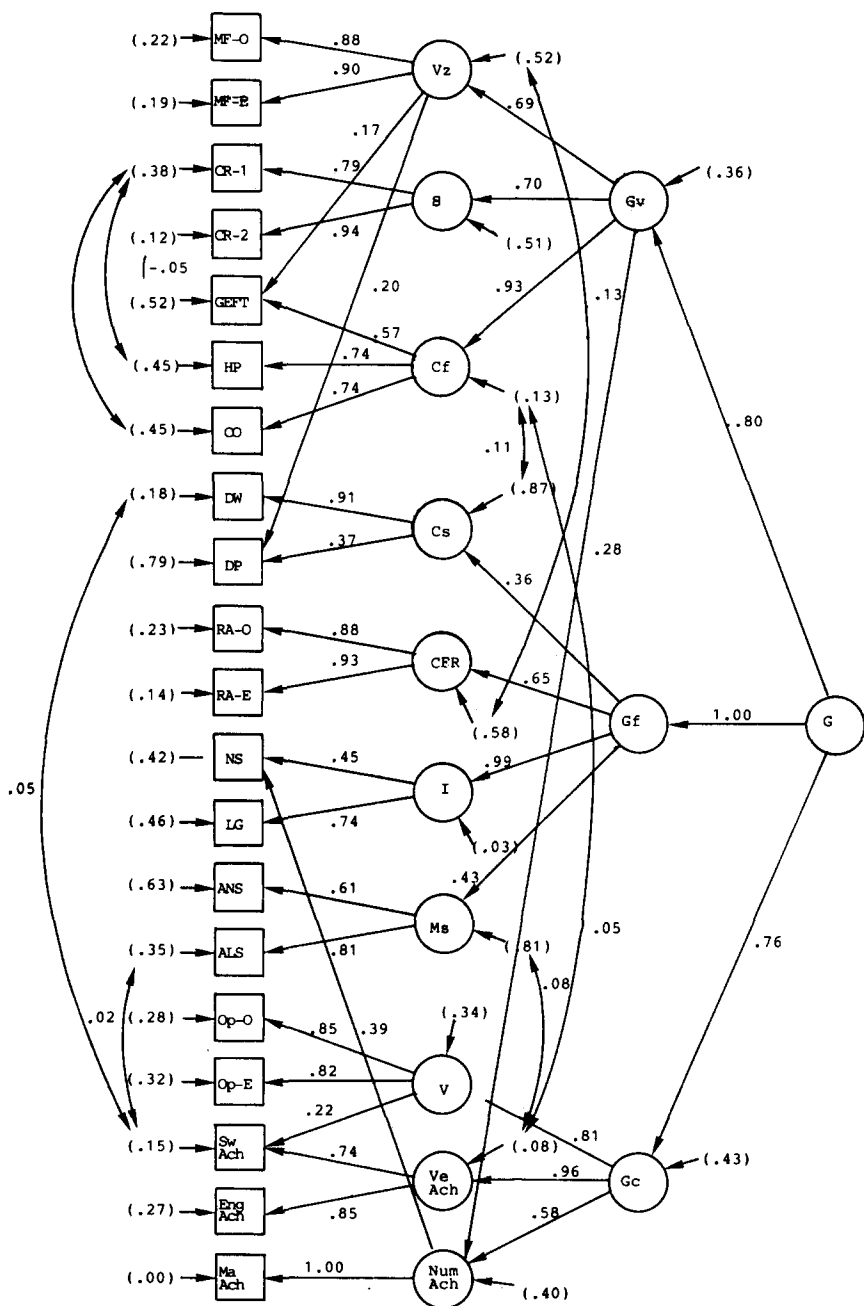


FIG. 1. The LISREL model with 1 third-order and 3 second-order factors.

interpreted as standardized factor loadings. The curved bidirectional arrows indicate correlation, without any assumptions about causality. Estimates of parameters are also presented in the figure, with the numbers in parentheses expressing estimates of residual variances.

It may be noted that Cf is the primary most highly loaded in Gv, and that the I-factor has a loading in Gf which approaches unity. Estimates of the I primary would thus be rather close estimates of the *g*-factor.

### *Discussion and Conclusions*

The LISREL models for the test battery provide strong support for the hypothesis that Gf is identical with *g*. It would, of course, be rash to conclude that one single empirical study proves the identity of Gf and *g*. However, other studies too have provided evidence along this line. Thus, Gustafsson et al. (1981) report a LISREL reanalysis of the Undheim (1978) study. In the reanalysis the 5 second-order factors hypothesized in the Cattell-Horn model were identified, and it was found that Gf had a loading of 1.00 in the third-order *g*-factor.

### THE HILI-MODEL

The fact that Gf seems to be identical with *g* makes it possible to construct a model of the structure of abilities which serves as a synthesis of most previously suggested models. This unifying model, which will be referred to as the HILI-model (Hieararchical, LISREL-based model), contains in its simplest version factors at three levels (cf. Figure 1): The first level contains the factors in the Thurstone and Guilford tradition; at the second level there are two broad factors, roughly covering performance in the verbal and figural domains; and at the third and highest level the *g*-factor is found.

It must be realized, however, that the lower-order factors in the HILI-model contain the specific variance, after the variance accounted for by the higher-order factors has been partialled out. To differentiate the first-order HILI-factors from the corresponding PMA-type factors, an ' is added to the former, e.g.,  $V'$ ,  $Cf'$  and  $Vz'$ . For the same reason, the second-order factors in the HILI-model are referred to as  $Gc'$  and  $Gv'$ .

The Spearman, Thurstone, and Cattell-Horn models may, in a structural sense at least, be viewed as subsets of the HILI-model: the Spearman model takes into account variance from the third-order factor; the Thurstone model takes into account first-order variance; and the Cattell-Horn model takes into account both first- and second-order variance. The Vernon model comes close to the proposed model: The *g*-factor is included in both models, and at the second-order level  $Vz'$  closely corresponds to  $Gc'$ , and  $k:m$  corresponds to  $Gv'$ .

The Guilford SI-model is the only previously suggested model which is clearly incompatible with the HILI-model. This is because the factors in the SI-model

are taken to be orthogonal, which precludes reduction of primary factors to higher-order factors. It appears that Guilford (1980) now, however, admits the possibility of correlations between abilities, which paves the way for a reformulation of the SI-model in hierarchical terms.

It must be pointed out, though, that until a broader range of abilities has been investigated, one should hesitate to draw strong conclusions about similarities and differences between the different models. Thus, the Cattell-Horn model includes the second-order factors *Gs* and *Gr* as well, which factors have not yet been included in the HILI-model. Vernon places tests measuring these factors below *v:ed*, so there clearly is a need for further research.

The open-endedness of the model must be stressed. It may, in fact, be more correct to talk about a class of HILI-models, than about the HILI-model. Models belonging to this class are hierarchical and include the basic set of higher-order factors shown in Figure 1. The number of levels in the hierarchy may differ from model to model, however, as may the domains of tasks which are sampled. Thus, when a certain area is represented by few tests there will necessarily be few levels in that branch of the hierarchy (cf. Cattell, 1966). When an area is investigated in great detail, as Fredriksen (e.g., 1980) has done with reading, for example, it may be possible to add levels both below and above what is here taken to be the primary level.

### *Interpretations of the Factors*

Higher-order factors are abstract entities which are not easily interpreted in psychological terms. Some suggestions have been offered, however, for interpretations of these sources of individual differences, and these are discussed next.

The *g* factor has been remarkably absent from theoretical discussions since the pioneering work of Spearman. At best it might be said that attention has now and then been called to the fact that the factor exists (e.g., Humphreys, 1962; McNemar, 1964). During the last few years, however, it appears that interest in the general factor has been revitalized, as may be seen in the writings of, for example, Humphreys (1979), Snow (1977, 1978, 1980), Sternberg (1980, 1981b), and Undheim (1981c).

As has already been mentioned, Undheim (1981a, 1981c) has proposed ideas almost identical with those espoused here, and from a series of studies it was concluded:

Although Cattell's hypothesis of two intelligence factors, fluid and crystallized intelligence, is seemingly supported by the simple-structure factor analytic distinctions of two such factors in several studies, hierarchical order analysis indicate that these findings may support an alternative hierarchical model of intelligence where fluid tasks are central in the definition of general intelligence and group factors of crystallized ability or verbal-educational



knowledge, visualization, and speediness emerge. Thus the results are consistent with a more parsimonious neo-Spearman structuring of broad ability factors (Undheim, 1981a, pp. 184–185).

This conclusion thus conforms with the conclusion drawn here.

In his interpretation of general intelligence Undheim (1981c) stressed that *g* is a consequence of learning, and that the nature of intelligence is determined by cultural values: “general intelligence is good reasoning with the contents of our culture” (Undheim, 1981c, p. 256). This line of reasoning led Undheim to suggest a very broad definition of general intelligence, namely, that it represents the entire repertoire of knowledge, skills, and strategies. Undheim also concluded that:

a measure of general intelligence should sample achievements in many subject matters—some of which are tied to the academic curricula that subjects are exposed to, others tied to intellectual achievements acquired out of school (Undheim, 1981c, p. 257).

One problem with this “sampling” interpretation of *g* is that it disregards the fact that *Gf* has been shown to be identical with *g*. Formulated in simple terms this result implies that scores obtained on a test consisting of the broadest possible and most representative sample of tasks are virtually perfectly correlated with scores obtained on a small set of *Gf* tasks. The most interesting question must then be why the *Gf*-tests have such power of indexing general intelligence.

In speculations on the nature of *g* many authors have stressed that one important characteristic of tasks to measure *g* or *Gf* is that they present the examinee with new problems (e.g., Spearman, 1927, pp. 161–198; Horn, 1968; Cattell, 1971). Sternberg (1981b) has presented some empirical evidence that intelligence can best be understood through what he calls “nonentrenched” (i.e., novel) kinds of tasks.

Snow (1980) has gone one step further and has outlined a process theory of intelligence, in which theory to the novelty of tasks is seen as essential. Snow suggests that tests of general ability in particular may pose demands for new assembly of performance processes:

Perhaps they represent to a greater degree the kinds of assembly and control processes needed to organize on a short term basis adaptive strategies for solving novel problems. The more complex and varied the sequence of novel problems, the more adaptive the processing needs to be. The Raven Progressive Matrices Test is perhaps the archetypical example of such a test (Snow, 1980, pp. 35–36).

According to this interpretation, the most essential features of *g* tests are that they present novel and complex tasks. The novelty forces the examinee to find new ways of solving the tasks, and the complexity ensures that this is not simple: the

examinee must always be prepared to find new modes of attack, and with greater complexity follows that the number of steps and intermediary results to keep track of increases.

Snow's interpretation of *g* is admittedly abstract and vague but at the present state of knowledge it does seem quite impossible to carry interpretations any further. What is important, however, is that an interpretation is couched in such terms that it may be developed into more specific formulations, and from this point of view the Snow approach does seem profitable: it relates directly to flourishing research on information processing, computer simulation, and artificial intelligence.

Before leaving the discussion about the *g* factor it should be pointed out that proof of existence of such a factor does not provide support for other uni-factor conceptions. Thus, the construct of IQ is something quite different from the construct of *g*. IQ-tests are likely to be rather highly correlated with estimates of the *g*-factor, but in addition to *g* most such tests involve a substantial amount of *Gc'*.

It has already been hypothesized that *Gc'* is more or less identical with the *ved* factor of the Vernon model, even though there is as yet no empirical proof of this. Both factors, however, represent the verbal content area, with a strong leaning towards knowledge acquired through formal education, and both Vernon and Cattell-Horn mention tests of vocabulary as the best measure of the factor.

These factors also seem to be identical with a factor termed VEK (Verbal-Educational Knowledge) by Undheim (1981c). Undheim (1981b) argued that this factor may represent a rather narrow achievement factor:

it may be related to opportunity, interest, and effort in verbal-educational achievement in school—reflecting engaged time in school learning, in reading books more generally, reading newspapers and magazines, watching “educational” programs on TV, etc. (Undheim, 1981b, p. 186).

Thus, Undheim sees *Gc'* as being the accumulated result of choice of verbally-oriented activities. Such a theoretical position comes close to the “transfer” theory proposed by Ferguson (1954), and is supported, for example, by findings that choice of educational and occupational tracks do affect the relative strength of verbal and spatial abilities (Balke-Aurell, 1982).

Snow (1980) also has proposed that *Gc* is the result of prior learning and argued that it:

represents the long term accumulation of knowledge and skills, organized into functional cognitive systems by prior learning, that are in some sense crystallized as units for use in future learning. Since these are products of past education, and since education is in large part accumulative, transfer relations between past and future learning are assured. The transfer need not be primarily of specific knowledge but rather of organized academic learning

skills. Thus Gc may represent prior assemblies of performance processes retrieved as a system and applied anew in instructional situations not unlike those experienced in the past . . . (Snow, 1980, p. 37)

A similar line of reasoning could be constructed to account for Gv. In addition, however, to interpretations of Gc' and Gv' in terms of prior learning it may be that these factors reflect differential processing requirements of verbal and figural information. Thus, in the research on brain laterality (e.g., Bock, 1973; Harris, 1975; Nebes, 1974) it has been established that there are two broad modes of processing, one associated with the left hemisphere and verbal information, and the other with the right hemisphere and figural information. The first mode of processing is described as analytic, linear, binary, serial or successive, while the other mode is described as global, parallel, holistic, synchronous, simultaneous or continuous. It could thus be that Gc' and Gv' express the facility with which these types of processing, respectively, are performed.

It thus seems that two different explanations of individual differences in Gc' and Gv' may be proposed: one that takes its starting point in differences in long-term memory as a consequence of prior learning, and one that concentrates on the different processing characteristics of verbal and figural information. These interpretations are, of course, not mutually exclusive and they may both be true. There may also be quite intricate relationships between the two mechanisms. Thus, small initial differences in proficiency in a certain type of processing may affect interests and preferences, such that large differences in acquired knowledge result. It is also conceivable that availability of a large knowledge base enhances and expedites the type of processes which operate on that knowledge base.

There is little reason to discuss the wealth of primary factors at the lowest level in the HILI model. All the models considered here do include a smaller or larger set of narrow primary factors, and there is considerable overlap among the lists of factors, even though different labels may be employed (e.g., Guilford, 1972). In the HILI-model the primary factors represent the variance which is left after the variance from the higher-order factors has been partialled out. For many factors this is only a small fraction of the total variance, which residual in many cases may be of limited psychological interest. The theoretical position taken here is thus quite different from the one advocated by Detterman (1982), who argued that lower-order rather than higher-order constructs should be focussed upon in explanations of individual differences.

#### *Relationships with Nonfactorial Models*

The results which have been presented so far show that the HILI-model is compatible with most previous factorial models of the structure of abilities. It is interesting, however, to compare the HILI-model with other representations of the organization of abilities.

Snow (1980, see also Snow, Lohman, Marshalek, Yalow, & Webb, 1977) approached the problem with multidimensional scaling and hierarchical cluster analysis. Application of these methods to correlation matrices for large numbers of tests typically yield a scatter, where each test is represented as a point in two-dimensional space. At the very center of the scatter are tests of Gf, while tests of Gv and Gc appear as clusters not far from the center. In the more peripheral regions of the two-dimensional chart there are clusters of tests which may be identified with primary factors.

Marshalek (1977) interpreted the degree of centrality of a test as reflecting the complexity of the processing involved, or the involvement of *g*. Marshalek also pointed out that this model of intelligence is compatible with hierarchical models, such as Vernon's model: The degree of centrality represents the level in the hierarchy, and the clusters of tests represent factors. It would thus seem that this model based upon multidimensional scaling is compatible with the HILI-model.

Sternberg (1980) presented a model of intelligence, in relation to which claims of generality have been made. It is interesting, therefore, to compare the Sternberg model with the HILI model.

The "componential theory of intelligence" proposed by Sternberg is based on the concept of component, which is defined as ". . . an elementary information process that operates upon internal representations of objects or symbols" (Sternberg, 1980, p. 6). On the basis of function, components are classified into five different categories: meta-components, performance components, acquisition components, retention components, and transfer components. Meta-components "are higher order control processes that are used for executive planning and decision making in problem-solving (p. 7), while performance components represent processes actually used in task performance.

Sternberg also classifies components according to level of generality into three categories: general components, class components, and specific components. General components are processes used in all tasks within a given universe; class components are processes used within a subset of tasks; and specific components are used in the accomplishment of single tasks.

This classification of components is utilized in an assumed hierarchical organization of tasks. For each task in a hierarchy the same general components are used, and for each task different specific components are used. The level in the hierarchy at which a task is placed is determined by the class components: tasks at the lowest level each require one set of class components, while tasks at higher levels require all the class components of tasks at lower levels within the same branch of the hierarchy.

Sternberg confronted several models of the organization of human abilities with this componential conception of task performance. With respect to Spearman's Two Factor theory it was argued that the *g* factor comprises a set of general components that is common to a wide variety of tasks, while the *s*-factors components have a much higher proportion of general components among them,

since for almost every task executive routines for planning and monitoring performance must be invoked. It was, thus, concluded that "individual differences in meta-componential functioning will be primarily responsible for the appearance of individual differences of a general nature" (p. 10).

The Thurstone PMA's were by Sternberg interpreted to reflect individual differences in class components, while the correlation among the primary factors is accounted for by general components. As an example, Sternberg mentioned the I-factor, which was argued to involve a relatively small set of class components (i.e., inference, mapping, application, and justification).

The concepts of fluid and crystallized intelligence were also discussed. Tests of Gc were interpreted to reflect "the products of acquisition, retention and transfer components, whereas fluid ability tests seem to involve the execution of performance components."

Sternberg concluded that "factor theories of intelligence are all right almost. What this means is that almost all factor theories of intelligence are right in the sense of being special cases of a more general psychometric theory, but that they are not quite all right when considered in isolation. They need to be complemented by componential theories . . ." (p. 12).

While there is no need to challenge the conclusion that componential theories are complementary to factorial models, it would not seem that the theory outlined by Sternberg is able to function as a super-theory, within which the different models of the structure of abilities are contained as special cases. This may be seen if the specific interpretations proposed by Sternberg are scrutinized.

Sternberg argues that the *g* factor in Spearman's Two Factor theory represents individual differences in meta-components; that the Thurstonian *I*-factor represents individual differences in a set of performance components; and that *Gf* reflects individual differences in the execution of performance components generally. It has been shown, however, that *g* is identical with *Gf*, and the empirical evidence also indicates that *I* is virtually identical with these higher-order factors. Sternberg thus proposes three different explanations for the same individual difference variance. Even though these explanations are not mutually exclusive, this indicates that the componential theory is much too loose to function as a general psychometric theory.

Even more important is the fact that while the factorial models identify and structure systematic sources of individual differences at different levels of generality, the componential theory models performance on intellectual tasks. This very fundamental difference between the factorial and componential approaches is seen if the content of the hierarchies of the two models is scrutinized. In the componential theory the hierarchy is a hierarchy of tasks, while in the factorial approach it is a hierarchy of sources of individual difference variance. This difference in focus of attention makes the factorial and componential approaches complementary, but it also implies that the componential approach cannot provide a theory under which the factor-analytic models may be subsumed.

It is, finally, interesting to consider the reasons why it has for so long been

possible for several models to coexist, which, at the surface at least, provide so different accounts of the structure of human abilities. Sternberg (1981a) attributed this to the fact that exploratory factor analysis is successful in supporting almost any model. At a more concrete level, however, this may be explained by differences among the exploratory techniques, and particularly in the way they deal with variance from general factors. It is well known that while some techniques readily produce a strong general factor, others cannot even be forced to indicate the presence of a general factor.

It may easily be demonstrated (e.g., Gustafsson et al., 1981; Humphreys, 1979) that when Multiple Factor analysis is used with orthogonal rotation, the general factor is "rotated away," by being represented as small positive loadings in all factors. However, in interpretations of factor analytic findings, loadings lower than .30 are rarely attended to, and often not even presented. It may thus be claimed that orthogonal rotations to simple structure are quite deceptive in the presence of a general factor.

If an oblique rotation is carried out, the general factor is represented as the correlation among the factors. There are two problems inherent in oblique rotations, however. One is that there are almost always small positive loadings scattered in the matrix, which cause the true correlation among factors to be underestimated. The other problem is that most oblique rotational methods allow the researcher to determine the degree of obliqueness of the solution: In the Promax method (Hendrickson & White, 1964) this is governed by the parameter  $k$ ; in the indirect oblimin method (e.g., Harman, 1967, pp. 325–326) obliqueness is governed by the parameter  $\gamma$ , and so on. Oblique rotational methods can, therefore, not provide "objective" empirical information on the amount of actual correlation between factors, and thereby not information on the importance of higher-order factors. In confirmatory factor analysis, however, all these problems are avoided.

The LISREL formulation of confirmatory factor analysis brings other advantages as well. Within LISREL the HILI model may be used as a so-called measurement model in investigations of the relations between factors of ability and other variables. With such models it is possible to study relations between the factors of ability and other variables, without the results being contaminated by error variance and specific variance in the psychological tests (e.g., Jöreskog, 1978; Gustafsson & Lindström, 1979). The fact that the model is hierarchical also makes it possible to formulate extremely parsimonious models for the relations with other variables, by invoking first the  $g$ -factor, and then invoking only as many of the lower-order factors as may be necessary (for an example of such an application, see Gustafsson, 1982).

In most studies it will be impossible to include tests to represent all factors and all levels in the model. However, even in those cases when relatively few tests are used the hierarchical approach and mode of thinking may be utilized, and the factors may be interpreted within the framework of the HILI model. For exam-

ple, if in a study interest is centered on the  $g$  factor a selection of three or four tests representing e.g., Gc, Gf and Gv may yield one common factor. This factor should come very close to the third-order  $g$  in the HILI model. The results in such a study may thus be compared to results obtained in another study with a much larger test battery, even though it will of course not be possible to separate error variance, test specificity, and the residuals of primary and second-order factors. The HILI model thus provides a framework for relating results obtained in studies in which different tests have been employed.

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